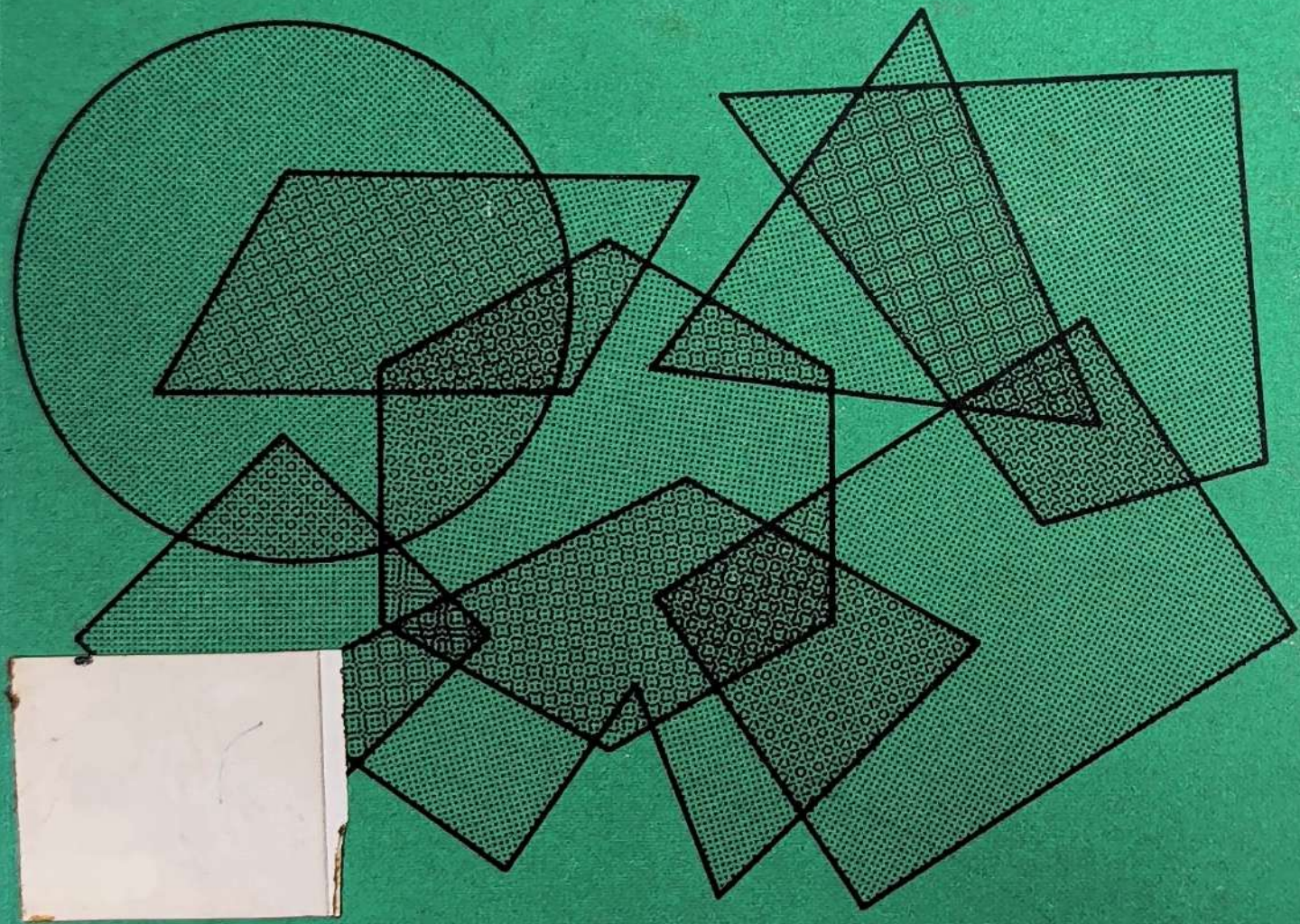
L. G. W. SEALEY

THE SHAPES OF THINGS



BASIL BLACKWELL · OXFORD

PZ
10
.S396
Sh3

THE SHAPES OF THINGS

L. G. W. SEALEY

Diagrams by Bernard Crossland

BASIL BLACKWELL · OXFORD

Distributed By
SPORTSHELF
P. O. Box 634
New Rochelle, N. Y. 10802

170919

© Basil Blackwell & Mott, Ltd., 1965

First printed in 1965
Reprinted 1967

Printed in Great Britain for Basil Blackwell & Mott, Ltd.
by A. R. Mowbray & Co. Limited in the City of Oxford
and bound at the Kemp Hall Bindery

A NOTE FOR TEACHERS AND PARENTS

THIS is the second of several background books to accompany the series concerned with the creative use of mathematics in the junior school.

It is intended that this book should be read at home or form part of the class or school library. It is not a text book to be used in the mathematics lesson, but one through which a child should browse.

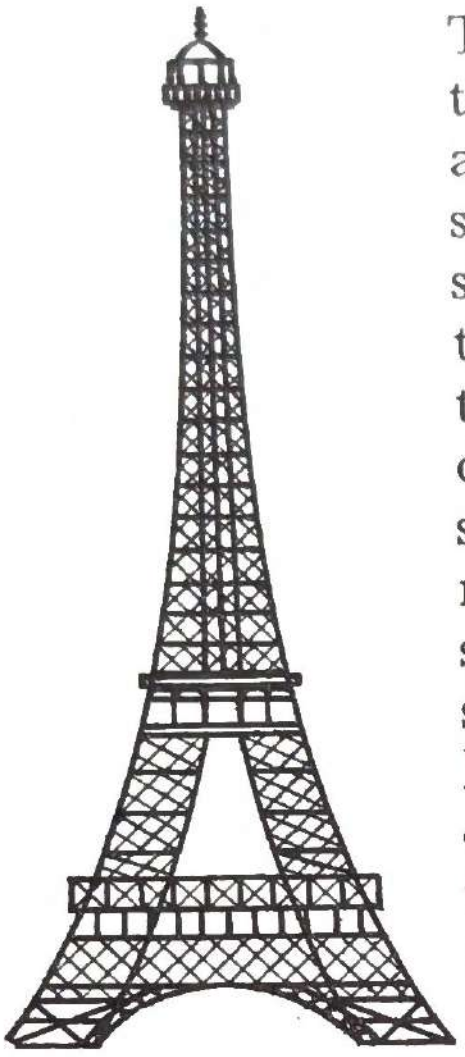
Some of the concepts introduced are sophisticated, but the approach has been to start with the child's environment and use reality as the basis for new thinking. No attempt has been made to deal with difficult concepts in depth.

CHAPTERS

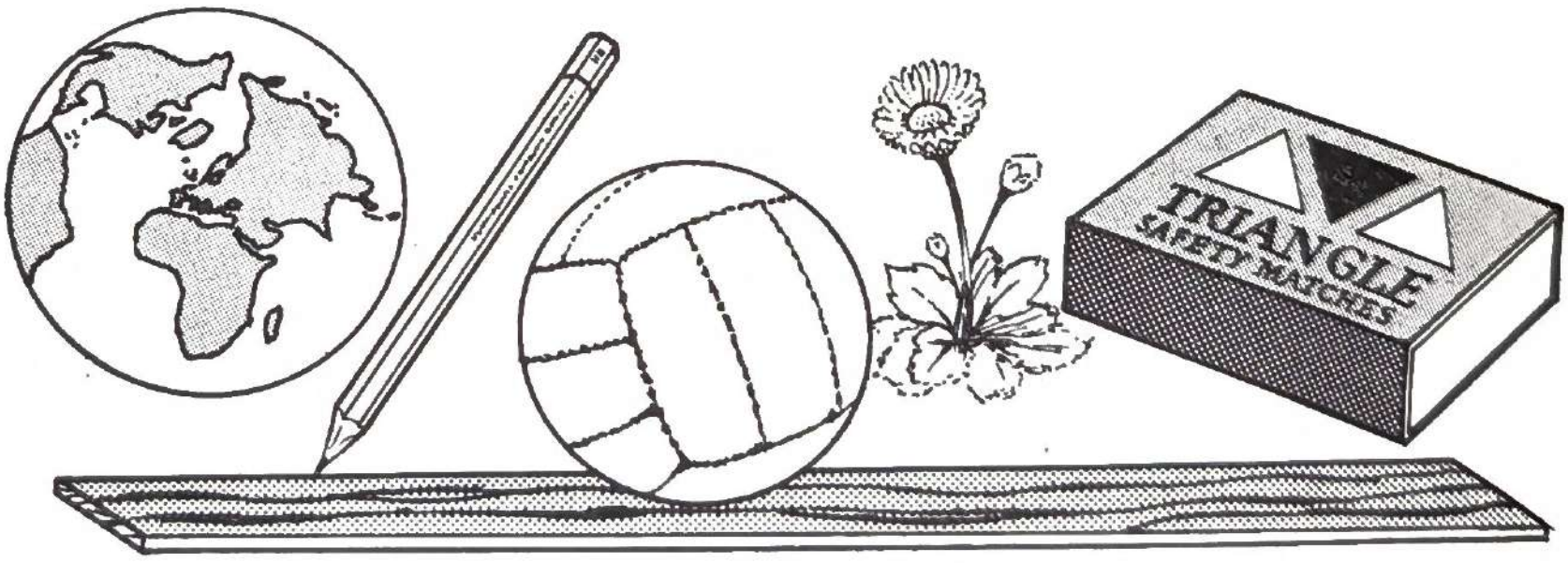
1. LOOKING AT SHAPES 2
2. WHY ARE THINGS THE SHAPES THEY ARE? . . 20
3. SOME NEW WAYS OF LOOKING AT SHAPES . . 36

CHAPTER 1

LOOKING AT SHAPES

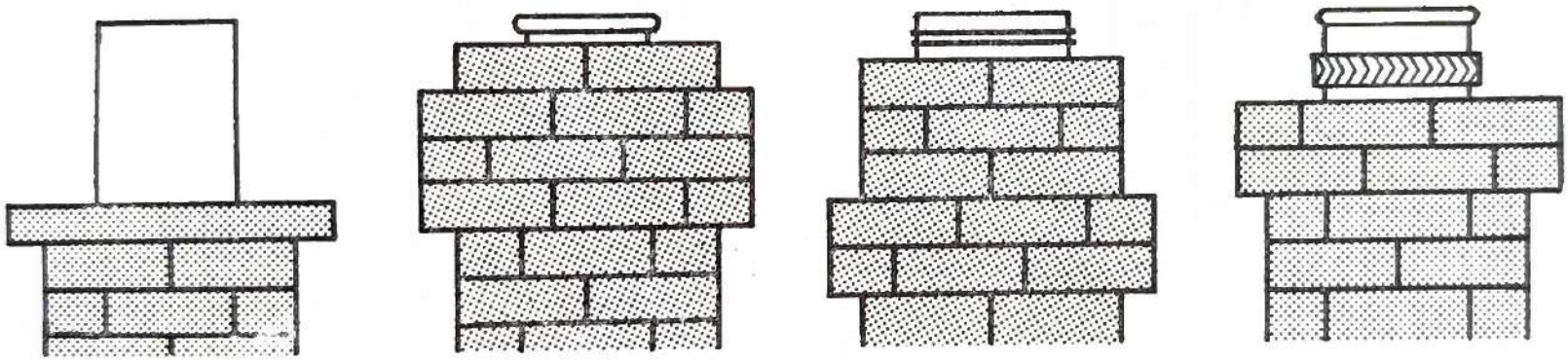


THE world is full of all kinds of things. Some are very large, such as the Eiffel Tower in Paris, and some are very small, such as a single ear of wheat. If you look at the two drawings you will notice that the shape of the tower is very different from the shape of the seed. The Eiffel Tower is a man-made shape. The seed is a natural shape. We can divide all the shapes that surround us in this way. Every shape will either be natural or man-made. Look carefully at these drawings and decide which shapes are natural and which are man-made.

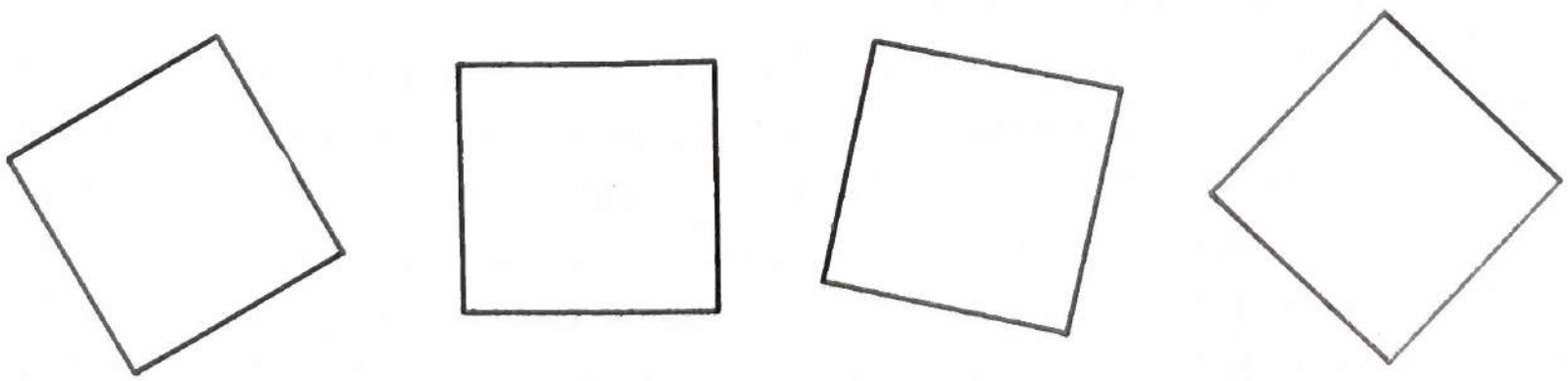


Notice how man uses the natural shape of the earth for the football. Notice, too, how we sometimes take natural materials, such as wood, and make them into shapes that are more useful. A growing tree is beautiful to look at, but the boughs and trunk of the tree are not good shapes for making into other things. Can you imagine the floor of your house made of boughs cut from the tree? Smooth and regular planks of wood are much better for this purpose.

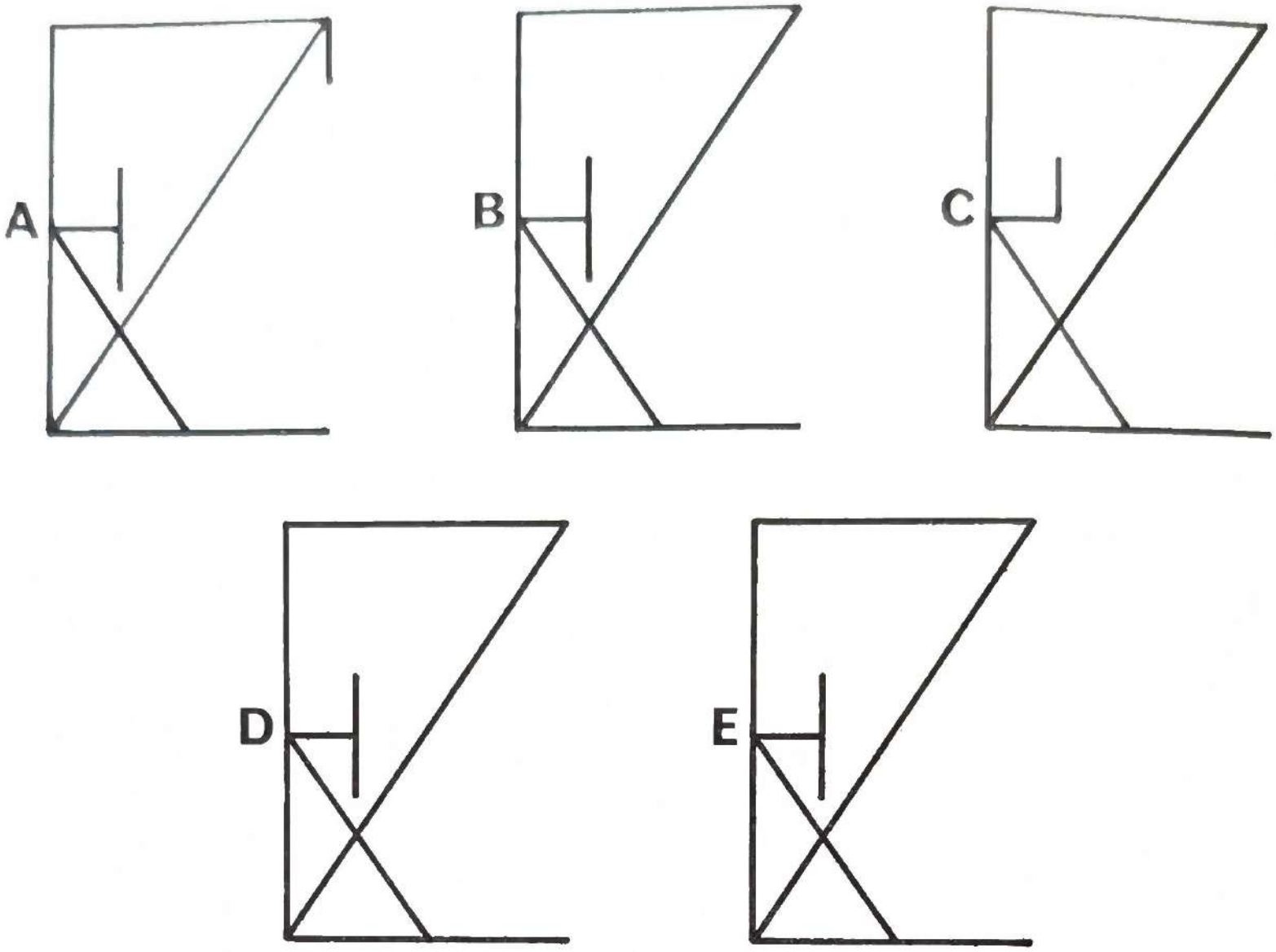
Some people do not seem to notice the shapes of the things around them. This is a pity, because the different shapes of things help to make the world more interesting. Even common objects such as the chimneys on houses can have very interesting shapes. Look at this collection of drawings of various chimney pots. All of these were seen from a window in the house of the artist who drew the pictures for this book.



It is easy to see the great differences between the shapes of the chimneys, but we need practice in spotting small differences. The shapes drawn below are all similar. They are squares of the same size but drawn in various positions. Sometimes we think things are different because their positions have changed.



Now look at these drawings:

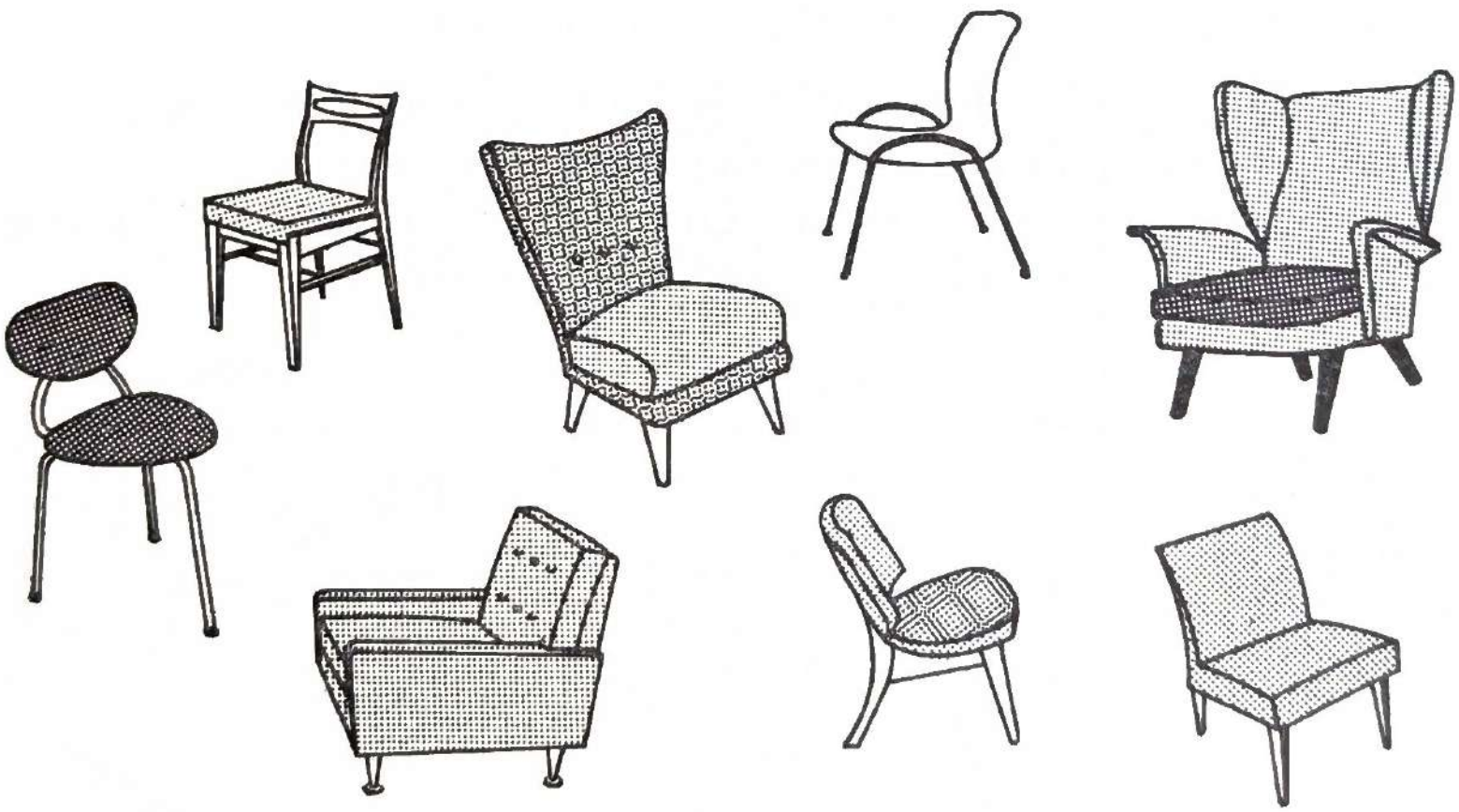


At first glance, all these shapes may look the same. Look at them very carefully. You will see that only the shapes marked B, D and E *are* the same. The other shapes are *slightly* different. Can you see the differences? It is good fun to make a series of drawings like this, and then ask your friends if they can decide which drawings are identical. To make the puzzle harder, the shapes may be drawn in various positions.

Have a good look at such things as the doors of the houses in your road. Even if some doors are of the same design every door will be different in some slight way. For example, one door may have a different letter box or handle. Another thing for you to do is to watch a moving object such as a car or a 'plane, and to notice how the shape seems to change as the position alters.

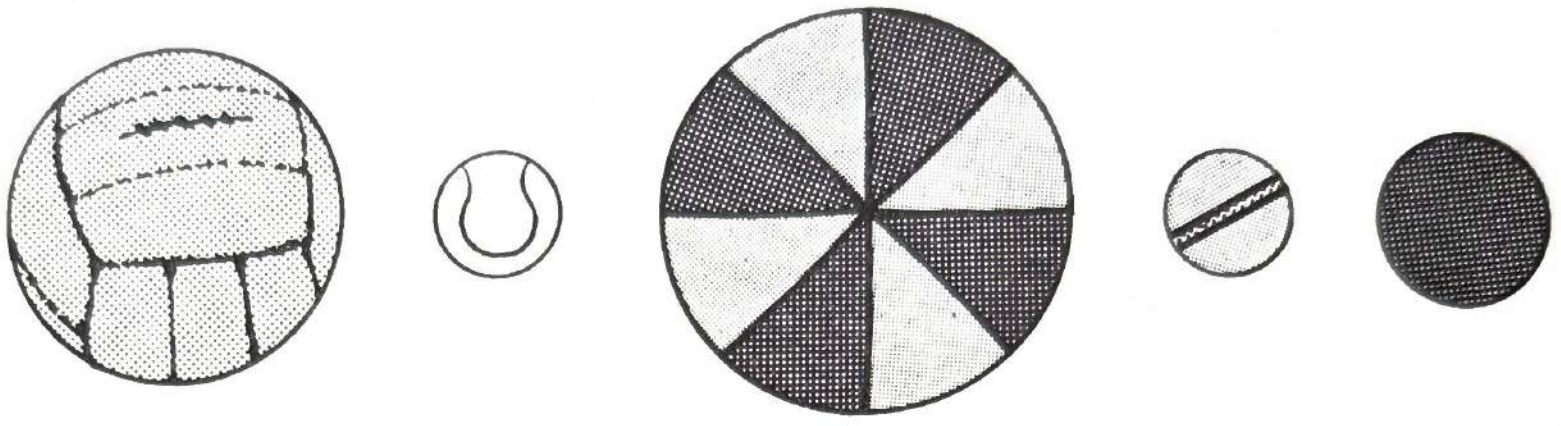
Some boys and girls become very clever at spotting cars or 'planes or railway engines, and can identify the various types at a glance.

Many of the shapes of the things around us are very complicated. Look at the shape of your hand. Study it carefully. Notice that the skin is not really smooth, but has bumps and hollows of all kinds. Notice how the shape of your hand changes as you move your fingers. Movement depends upon the use of muscles, and these expand and contract beneath the skin. Many of the things that we use every day also have complicated shapes. Study the shapes of the chairs that you have at home. The drawings below show some of the chairs that you might have.



How many different chairs can you see as you look around in your house and in the houses of your friends? The shapes of people's faces are also complicated. Shut your eyes and see if you can remember exactly the face of your best friend. It is rather surprising how young babies come to recognize people and the shapes of some of the things around them.

Let us take another look at a simple shape such as that of the football. Here are a number of balls of all kinds:



The balls are of different sizes, but they all have the same shape. We say that a ball has the shape of a *sphere*.

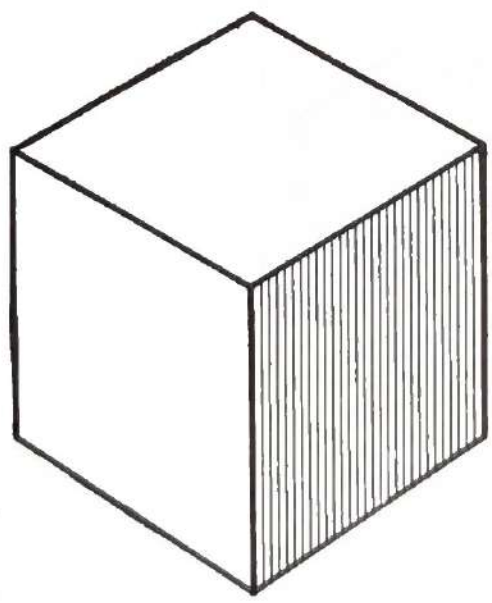
Here are some other objects. Our artist has drawn a die, a box containing a toy spinning top, and a biscuit tin.

Again, the things are of different sizes, but they all have the same shape. We say that each is a *cube*. If you look around you will find many other things that have the shape of a sphere or a cube.

When we talk of 'spheres' and 'cubes' we are using mathematical words. One important part of mathematics is the study of space and the things that are in space. By space we do not mean the great emptiness that surrounds the earth, the planets and the stars. There is space in the room in which you are sitting as you read this book. There are many things in this space. If your room is a classroom in school, there will be chairs and desks and other pieces of furniture. The study of spaces of all kinds and of the shapes of things of all kinds is called *geometry*.

A study of shapes must include a study of size. Shape and size seem to go together. We often tell our friends something about the size of an object when we are talking about its shape. So geometry includes the study of the sizes of things. The word 'geometry' comes from two Greek words. One of these words meant 'earth' and the other meant 'measurement'. The measurement of the earth was very important to people like the early Egyptians, and geometry was a very important part of mathematics thousands of years ago. If you are interested in this you will find other books in the library telling you more about it. To-day we do not think of geometry just as measurement of the earth. It is a much wider study, and still a *very* important one. This book is about the shapes of things, so it is really a book about geometry.

Look again at this larger drawing of a cube:

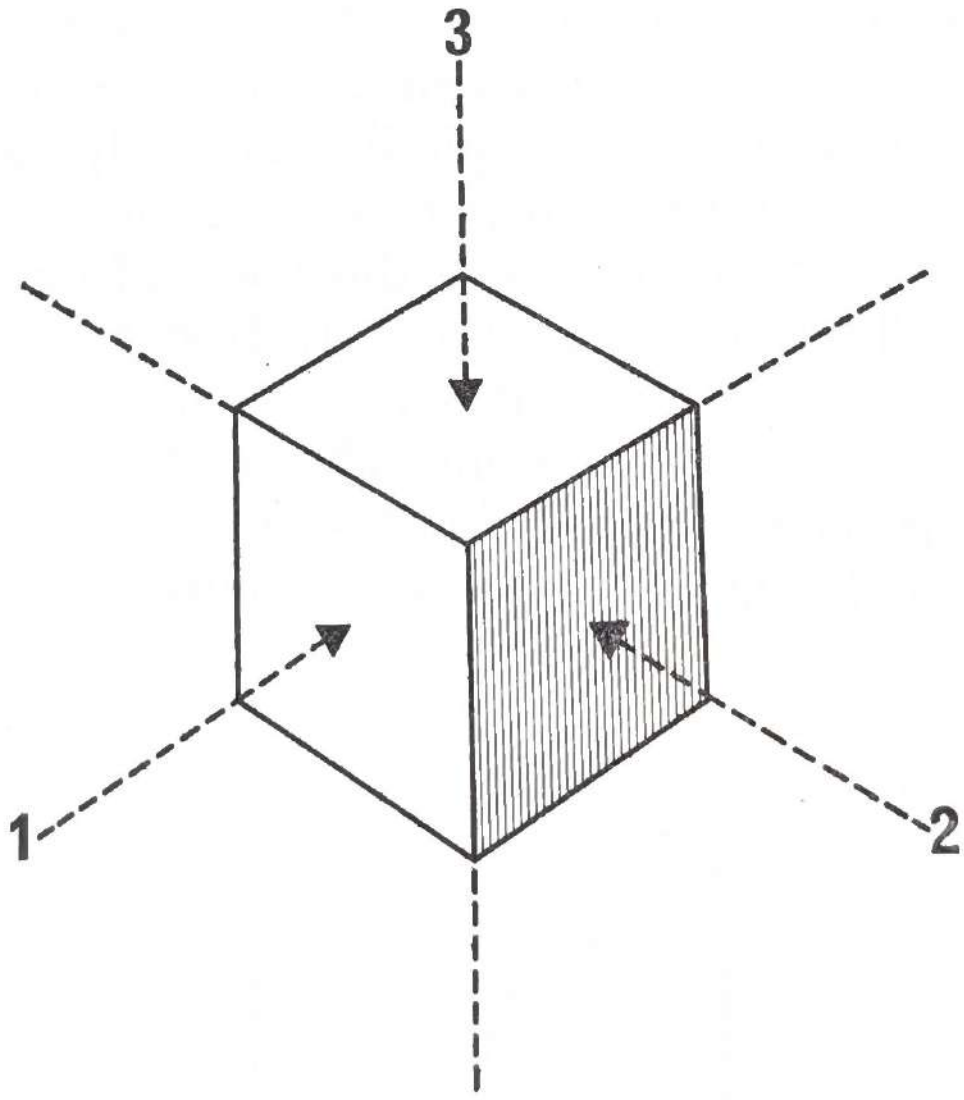


A real cube like this would be a solid object. You could hold it in your hand or stand it on a table. It is not easy to represent solid objects by drawings because the drawings are made on flat pieces of paper such as the page of this book. The real cube would have 6 sides.

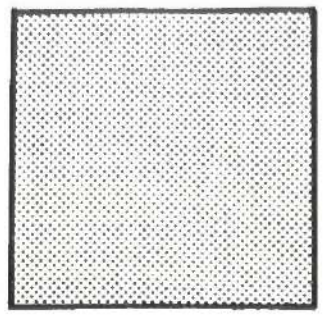
When we study the shapes of things we often look at the shapes of their sides. You can look at any object from many directions, but there are three main ones. You can look from:

(1) the front, or the back
(2) one side, or the other
(3) the top, or the bottom.

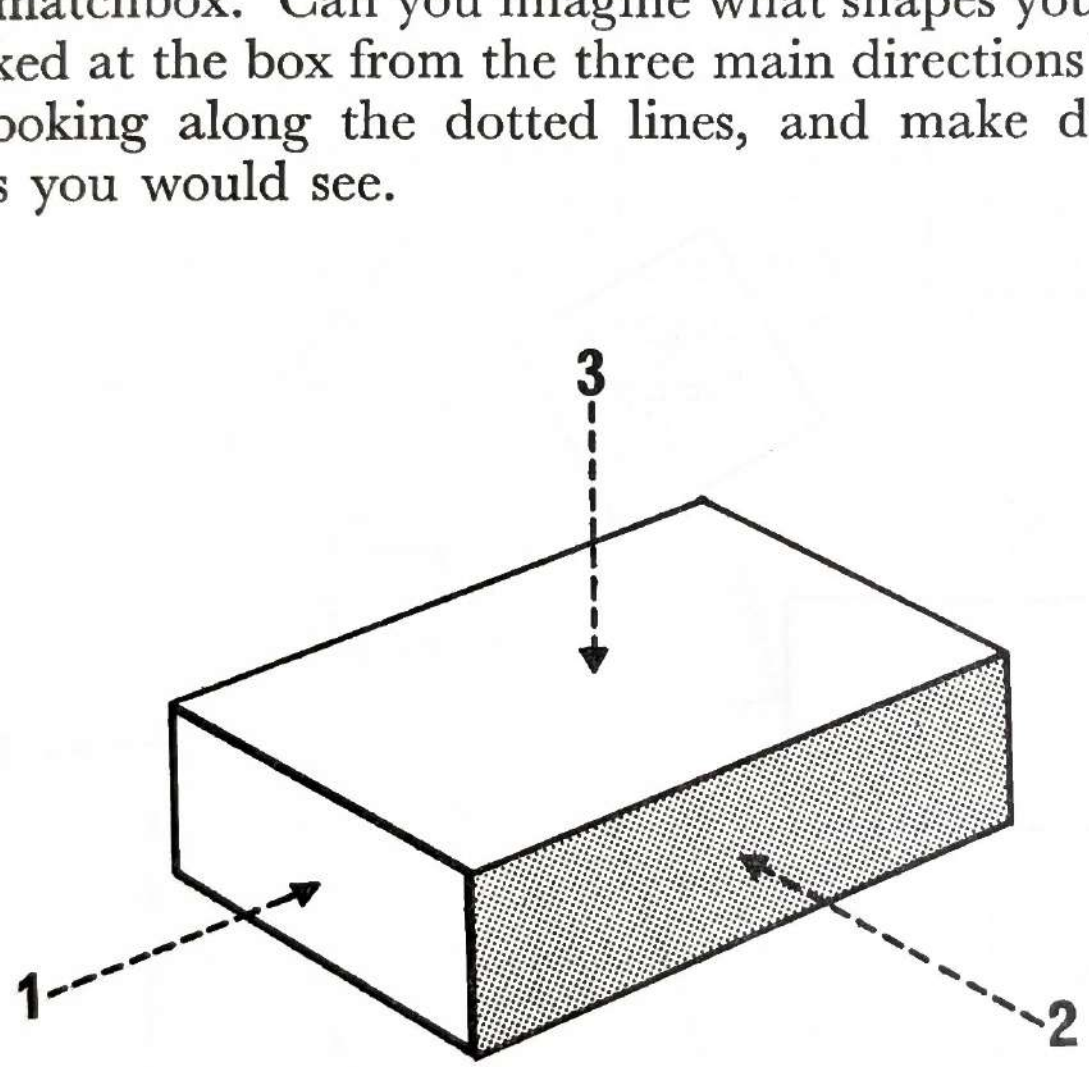
Here are the three directions drawn as dotted lines on the cube:



The strange thing about a cube is that the shape you will see if you look from any of the three directions will be the same! Each surface is a square and would look like this:



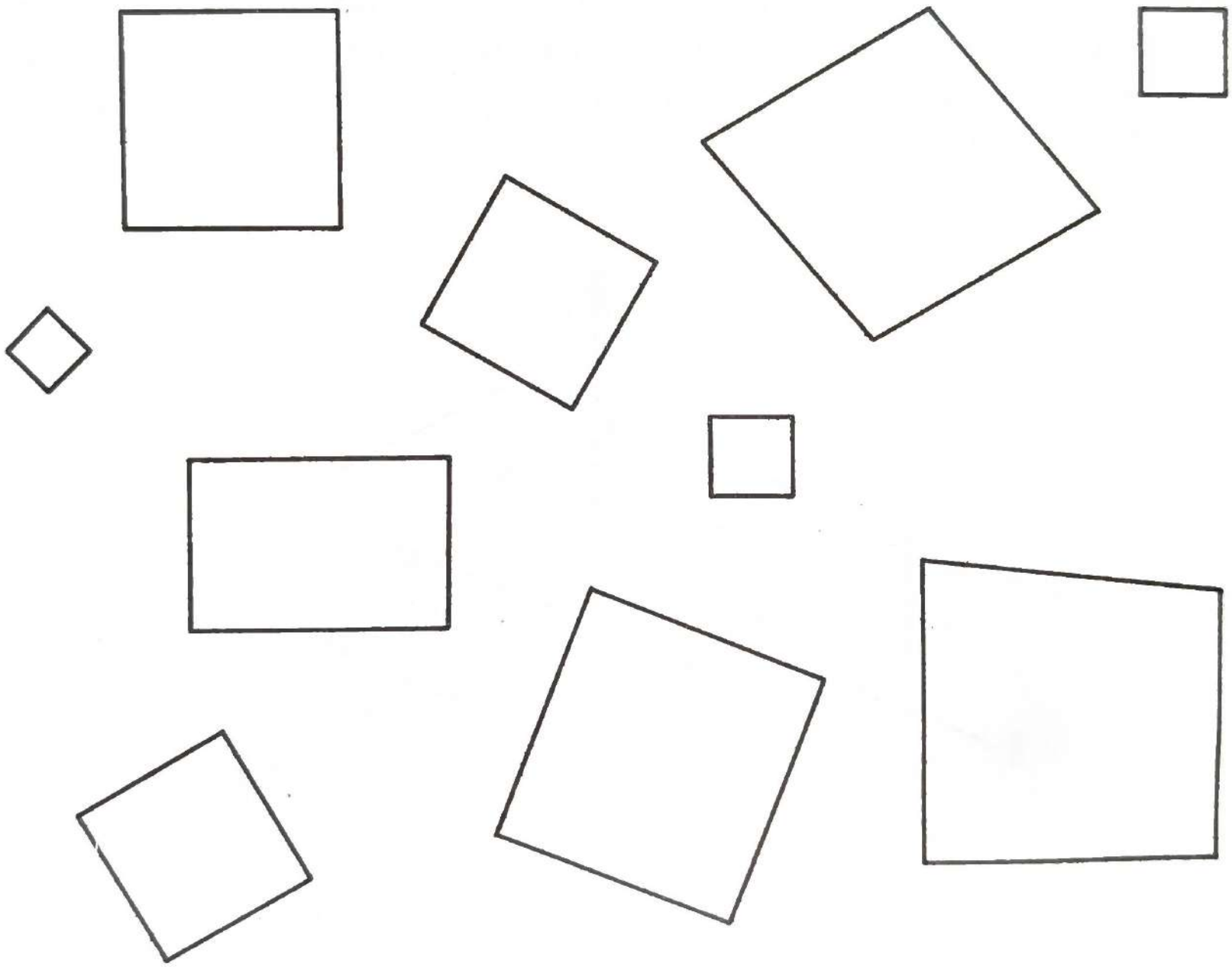
Of course, not many objects have sides that are the same. Here is a matchbox. Can you imagine what shapes you would see if you looked at the box from the three main directions? Imagine yourself looking along the dotted lines, and make drawings of the shapes you would see.



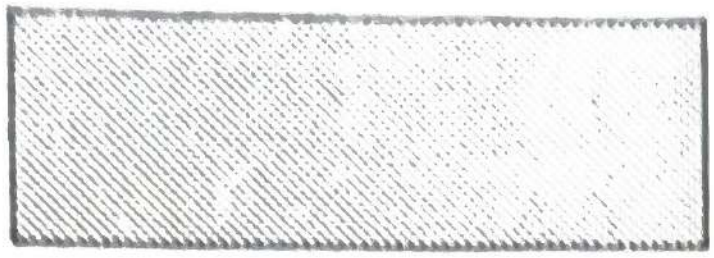
The view from the front of the matchbox is the same as that from the back. The shape of one side is also the same as the shape of the other. The top and bottom are also the same shape. But this is not so for all things. Can you find some objects where the views are different?

Some important Mathematical Shapes

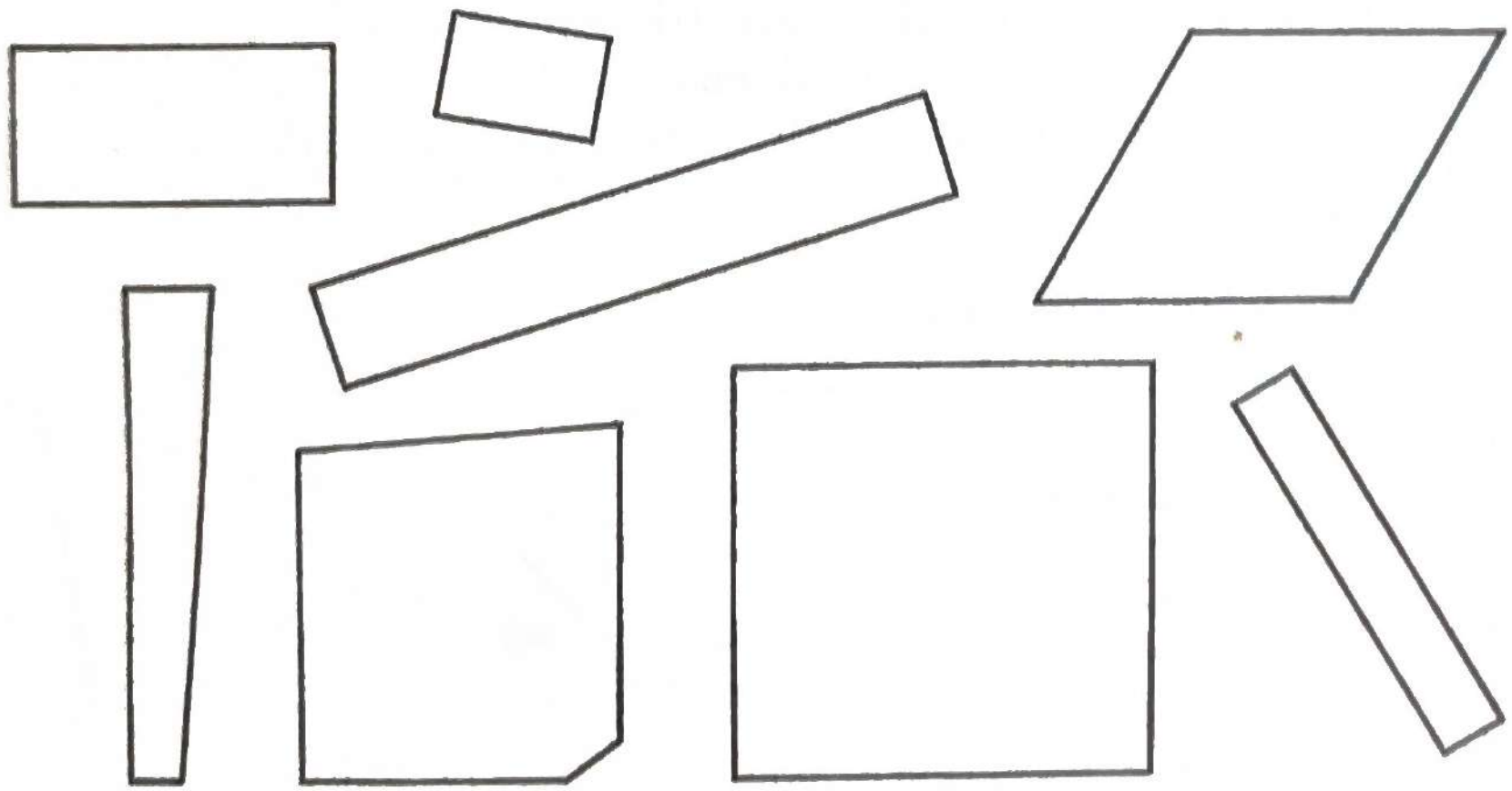
You know that the shape of the side of a cube is called a *square*. A square has 4 sides of equal length and these sides form 4 right-angles where they meet. For any shape to be a square it must have these two properties. All the drawings below except three are squares. Can you spot the shapes that are not squares?



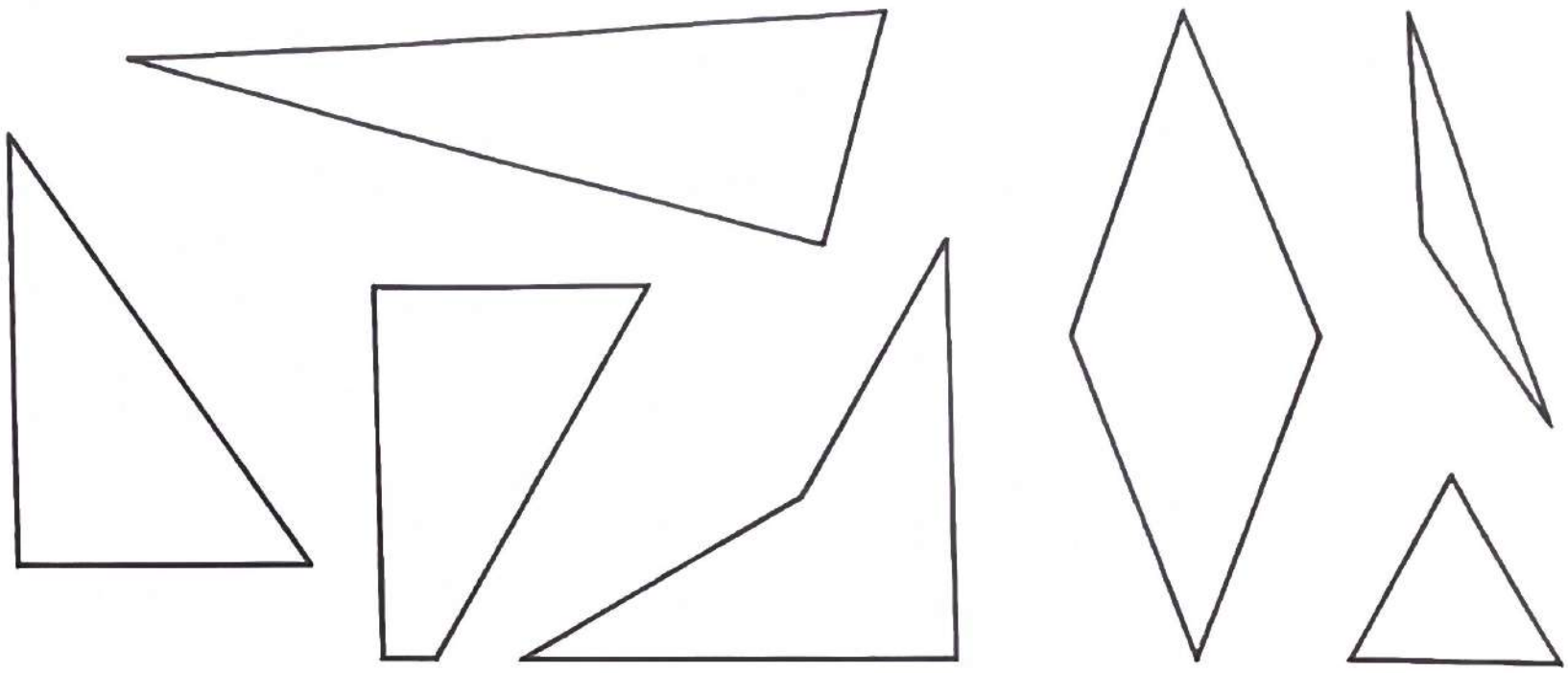
The sides of the matchbox are not square. Here is a drawing of one of them:



You will see that the shape has 4 sides and that these form 4 right-angles, but the 4 sides are not of equal length. Only the opposite sides are equal. One pair of opposite sides is longer than the other pair. A shape like this is sometimes called an *oblong*, but the more precise mathematical name is *rectangle*. Any rectangle must have 4 sides, with the opposite sides equal, and forming 4 right-angles. Look at the *square* again. It has 4 sides, 4 right-angles, and the opposite sides are equal! So a square is really just a special sort of rectangle. It is special because *all* the sides are of equal length. The drawings below are all rectangles except three. Can you spot the shapes that are not rectangles?

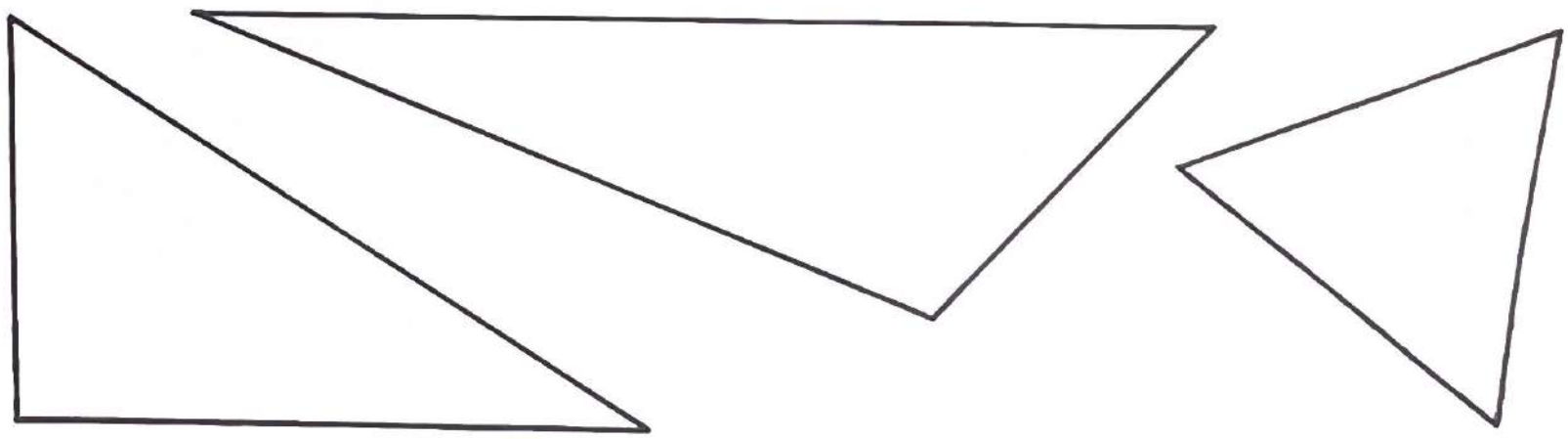


Another important mathematical shape is the *triangle*. The name 'triangle' will give you a clue about the shape. As you know, 'tri' means 'three', so triangles will be shapes with three angles. If a shape has just three angles it must have three sides. Can you see which of the shapes below are triangles? Remember that a triangle need not have sides of equal length or angles of any special size.

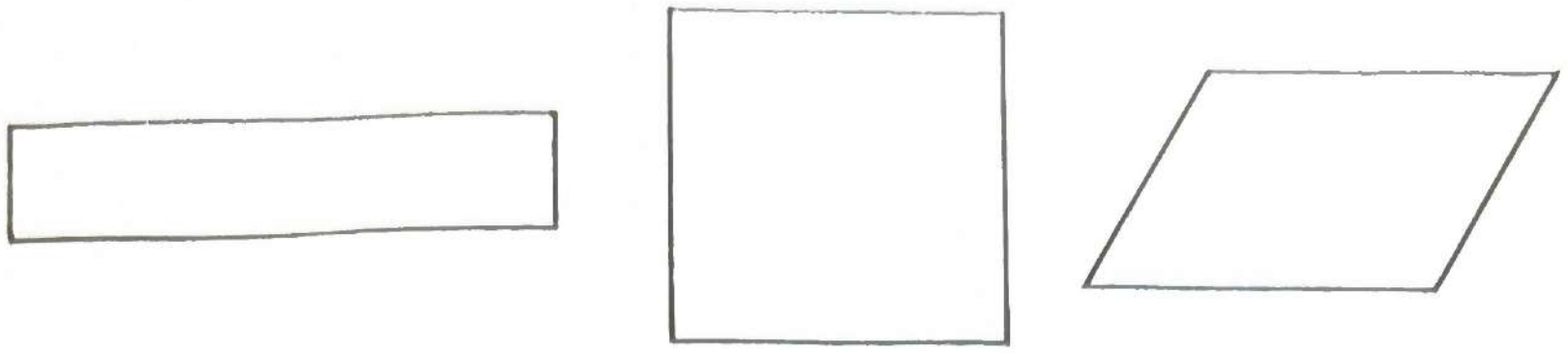


Squares, other rectangles and triangles have straight sides. There are other mathematical shapes that also have straight sides. One way of recognizing the shapes is to count the number of sides.

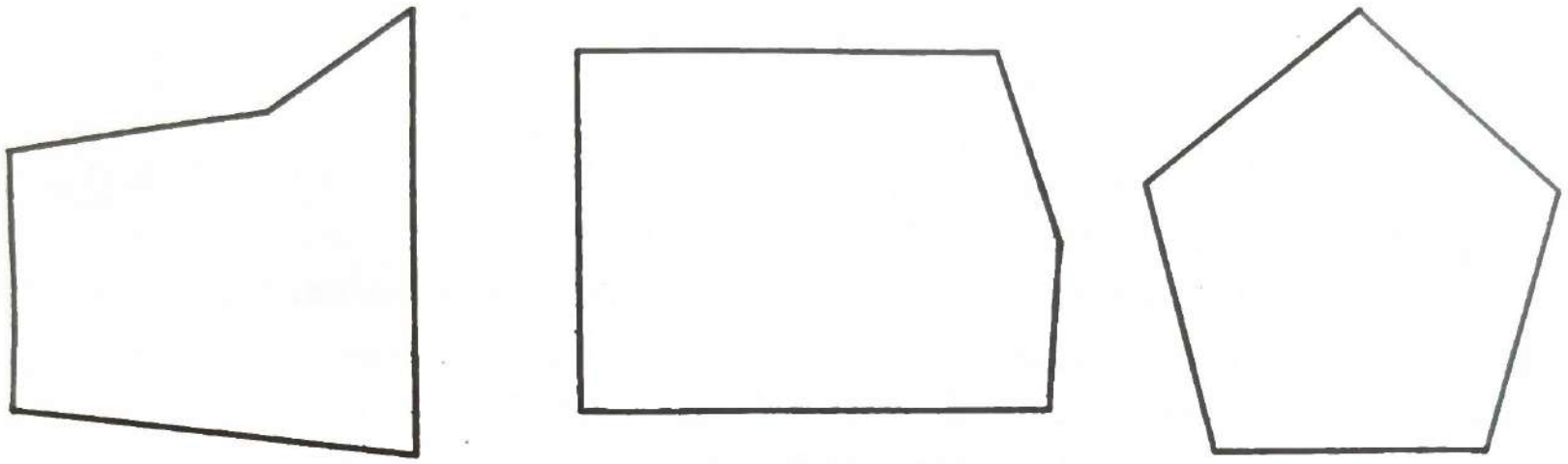
Triangles have 3 sides:



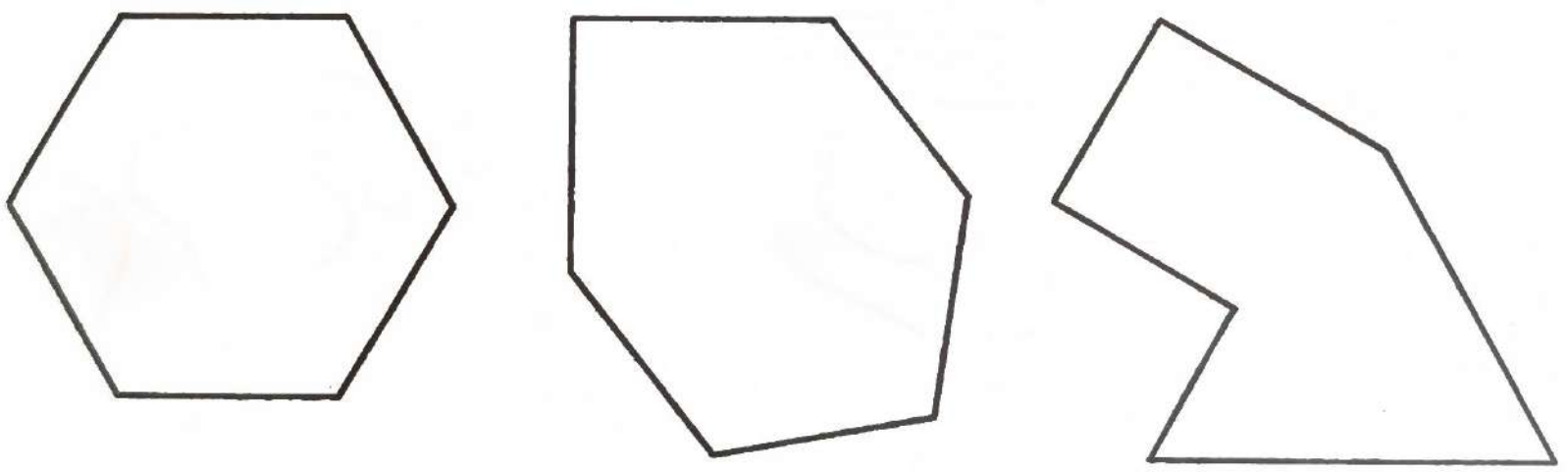
Quadrilaterals have 4 sides:



Pentagons have 5 sides:

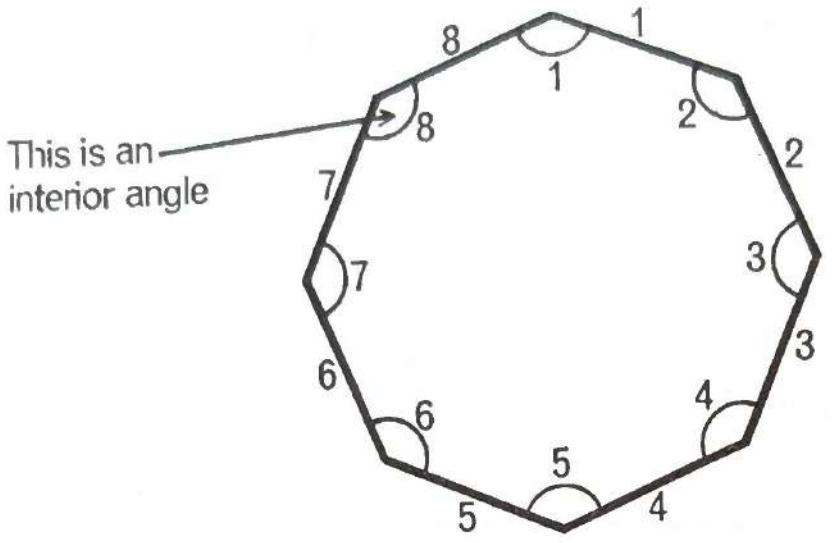


Hexagons have 6 sides:



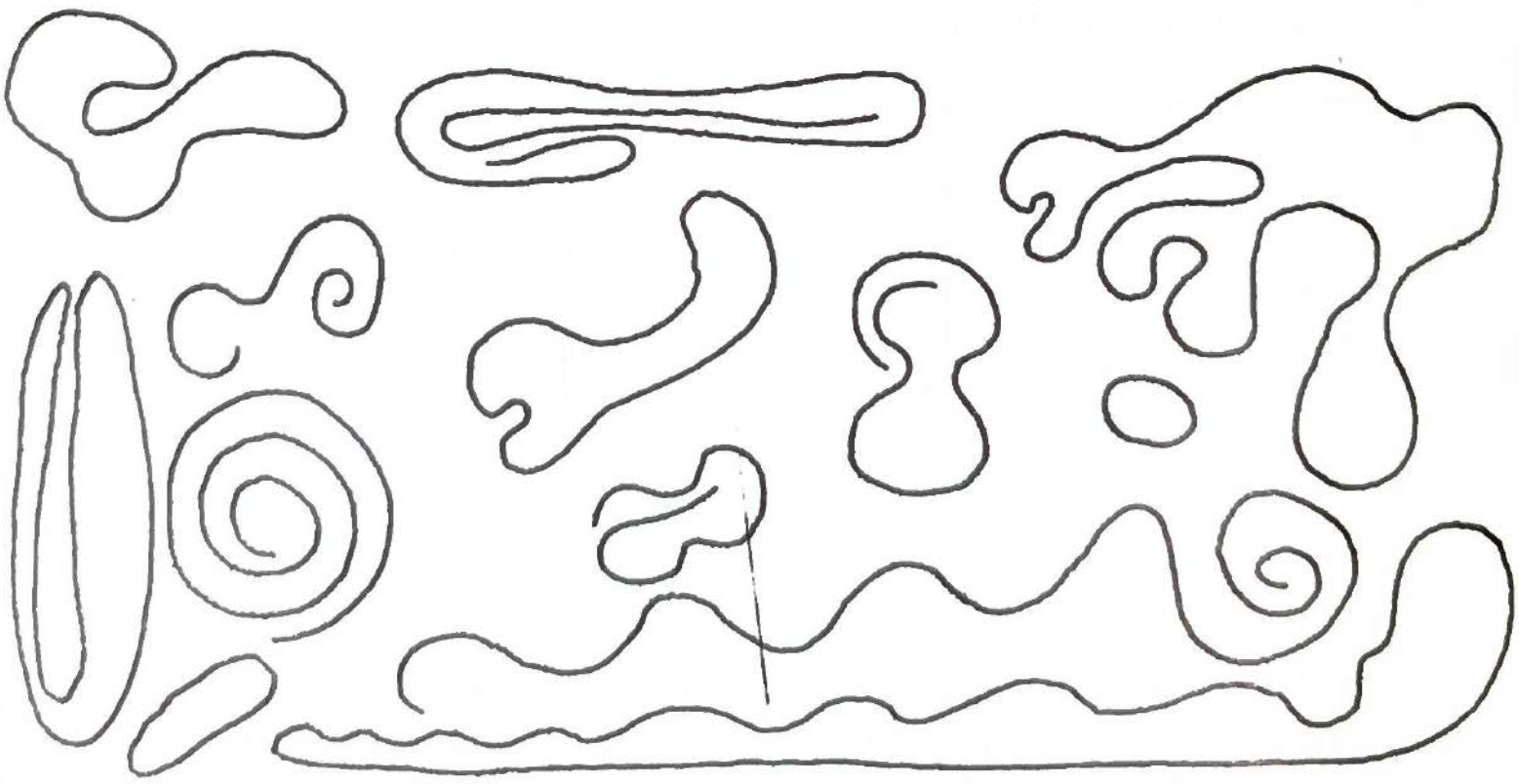
We could go on and on. Notice that the names 'pentagon' and 'hexagon' include 'agon'. This comes from the Greek word 'gonia' which meant 'angle'. So the name tells us that each shape has

the same number of interior angles as the number of sides. For example, an *octagon* has 8 sides and 8 interior angles:

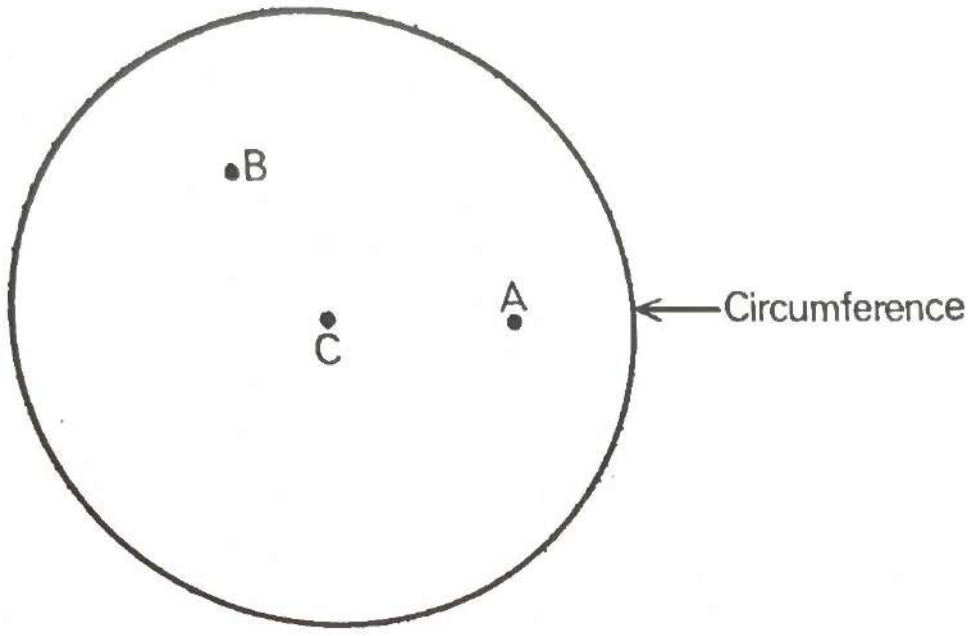


The general mathematical name for all these shapes is *polygon.* A polygon is a shape with many interior angles and sides.

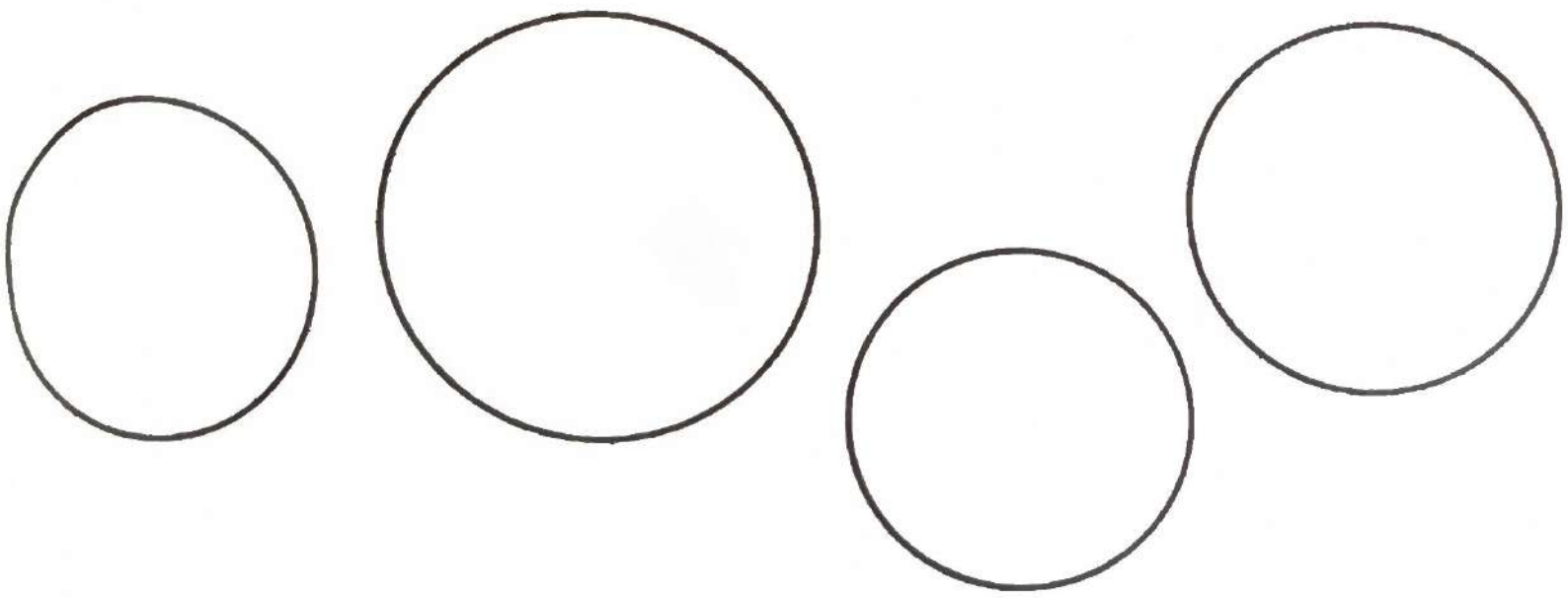
Not all mathematical shapes have straight sides. Two very important shapes that are formed by closed curves are the *circle* and the *ellipse*. A closed curve is one that is joined up. It is a continuous path. Here are some curves. Can you see which are closed?



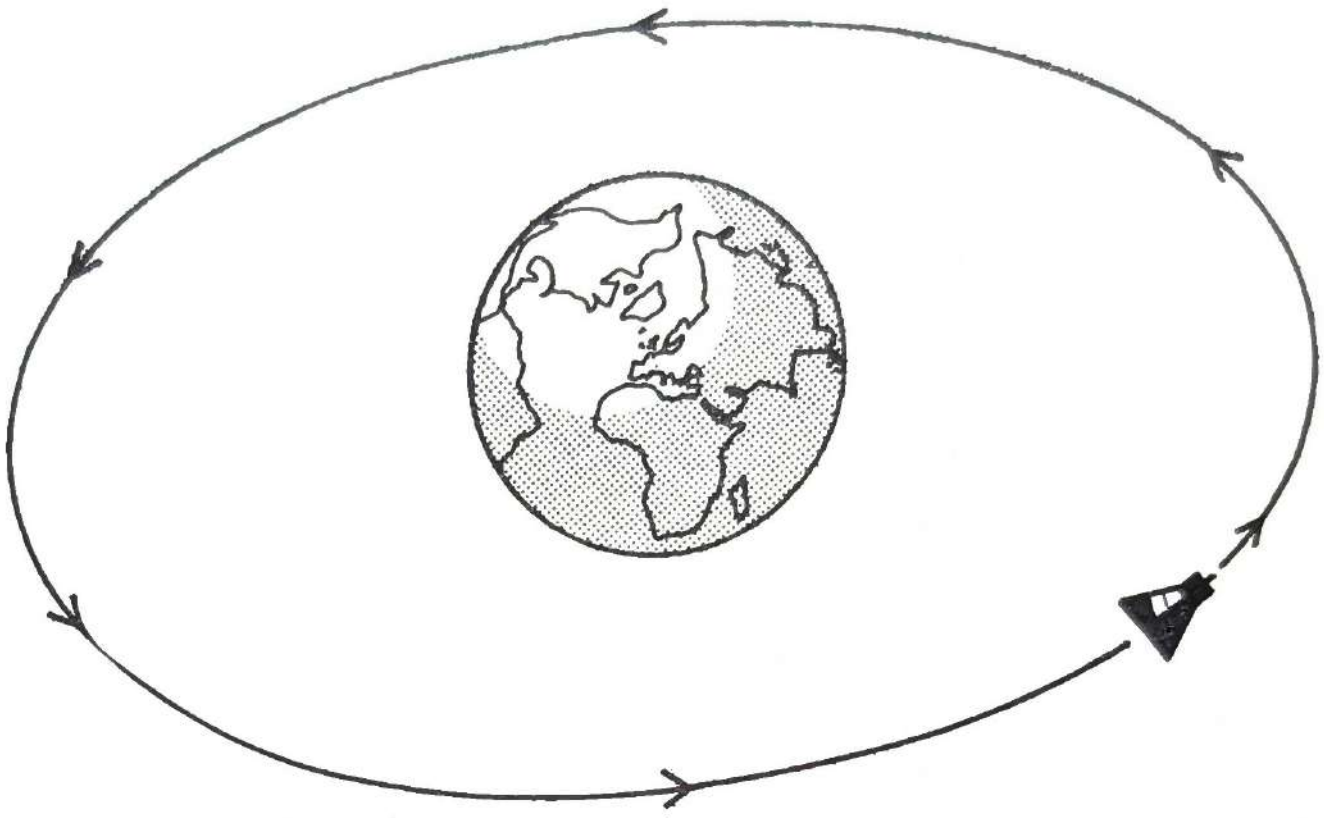
The circle is a very interesting shape. Inside every circle you can find a point which is equally distant from every part of the closed curve. Look at this circle and the points A, B and C marked inside it:



Points A and B are nearer some parts of the circumference than others. For example, point A is nearer the right-hand side of the circle than the left-hand side. But point C is exactly the same distance from *every* point on the circle. We say that point C is the centre of the circle. Can you find the centres of the circles below?

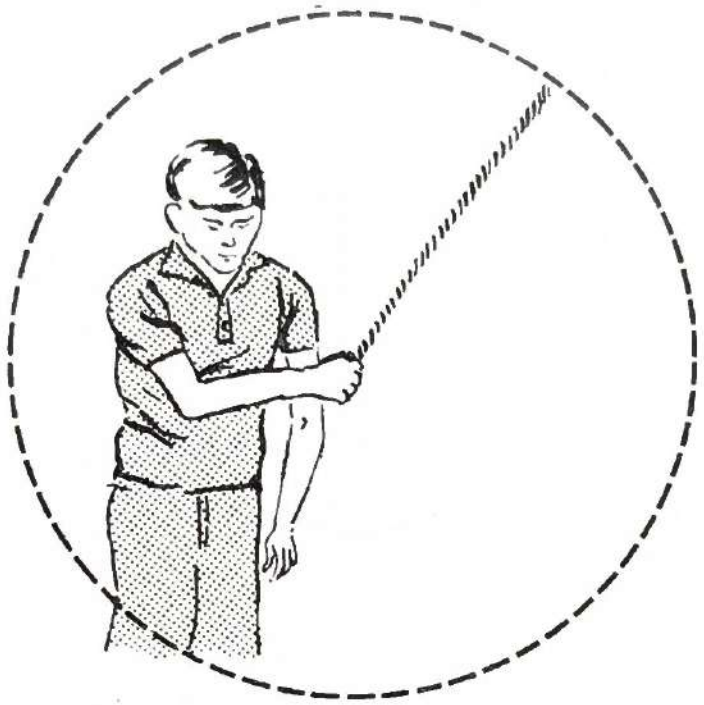


We hear quite a lot about ellipses these days. A satellite of the earth moves in a path the shape of an ellipse. Astronauts, as they speed around the earth, do the same.



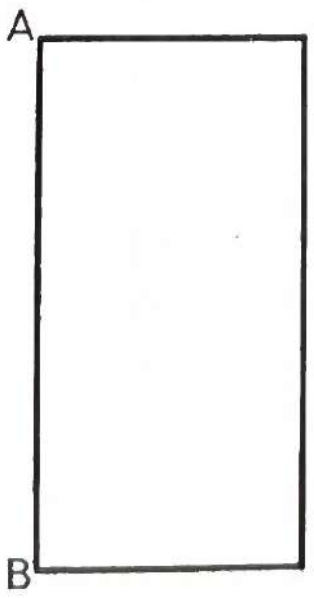
If you look at the drawings of the shapes we have been talking about, you will see that they are not *things*. Each face of the cube is a square, but you cannot pick up the square and hold it in your hand! If you take a ball and throw it into the air, the path that the ball follows from your hand and back to the ground again might look like this:

This curve has a special mathematical name. It is called a *parabola*. But you cannot pick up the parabola because it is just a shape—not a real thing. This boy is swinging a piece of rope round. The end of the rope follows a circular path as it twirls round. You would notice the circle but, again, the circle is not a thing.

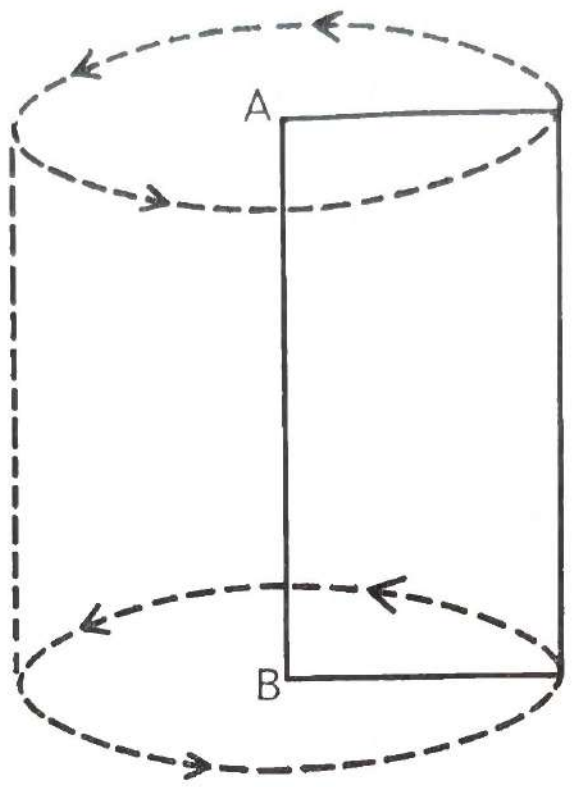


In mathematical language, we say that shapes of this kind are *plane figures*. They have no thickness. In real life, all the things around us have thickness. Even a very thin thing, such as the paper of this book, has some thickness. If it did not, then the book would not exist!

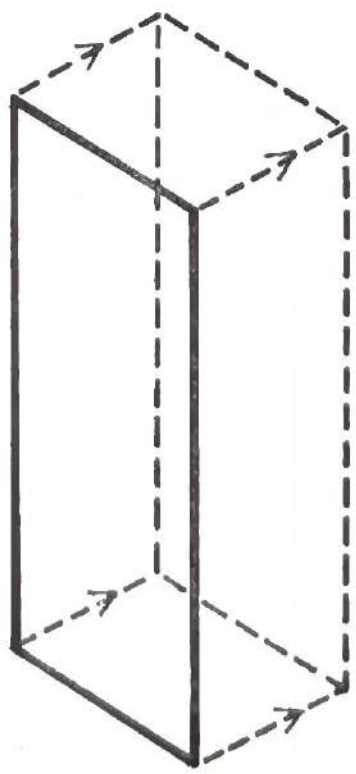
Let us see what happens if we take a plane figure and imagine that it has thickness. Look at this drawing of a rectangle:



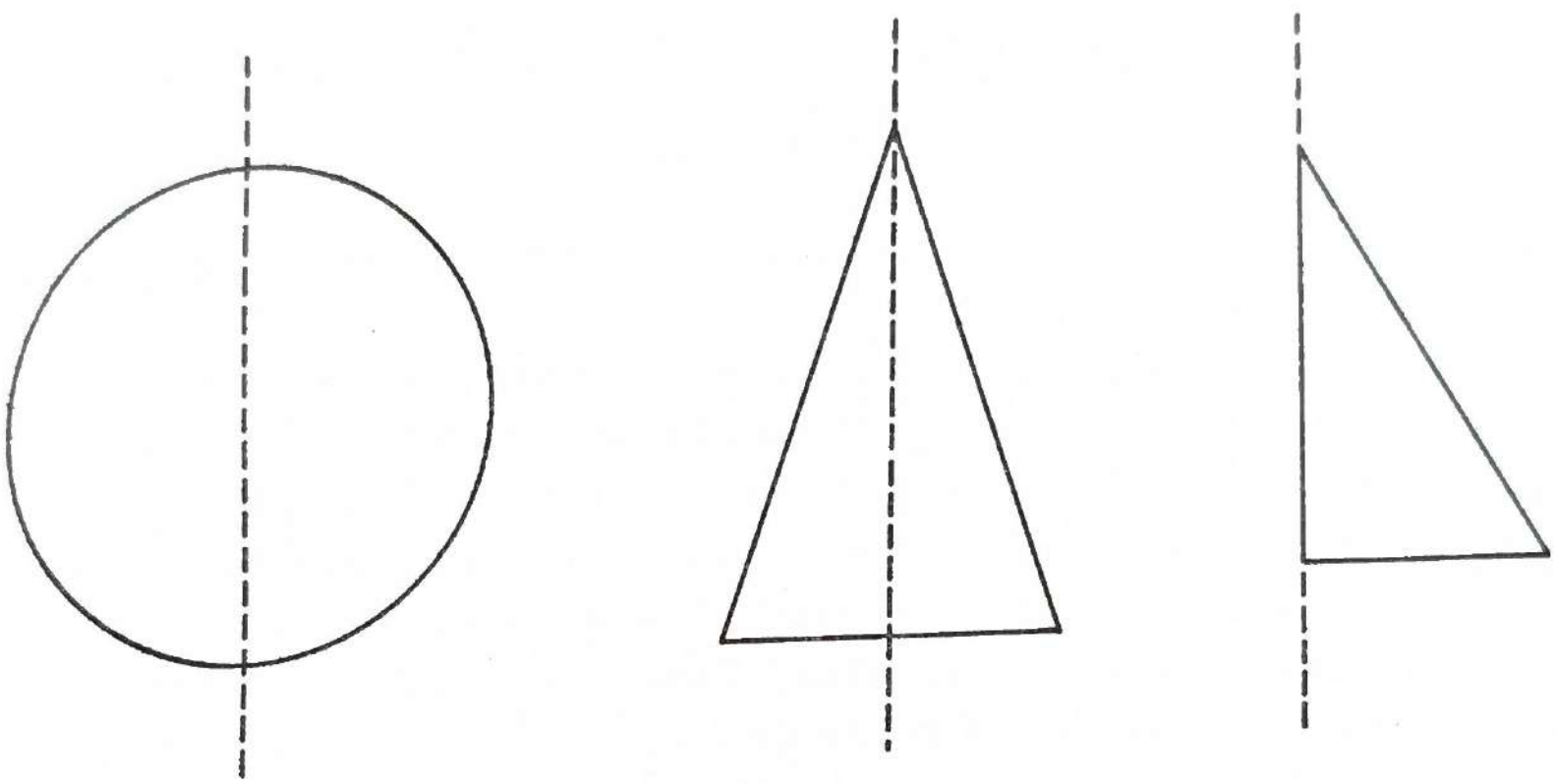
If the side of the rectangle marked AB was held quite still and the rectangle pushed round in a circle, we would have a *cylinder*:



Quite a different shape would be made if we pushed the whole rectangle backwards. The shape formed would look like a brick:



Can you imagine what shapes would be formed if you spun a circle, or a triangle? Look at the drawings below and try to think what would happen if the shapes spun round along the dotted lines.



All these are just some of the important mathematical shapes that you will see as you look around you at the world. Read the last two or three pages of this chapter once again and then try to find at least one example of all the shapes that have been mentioned.

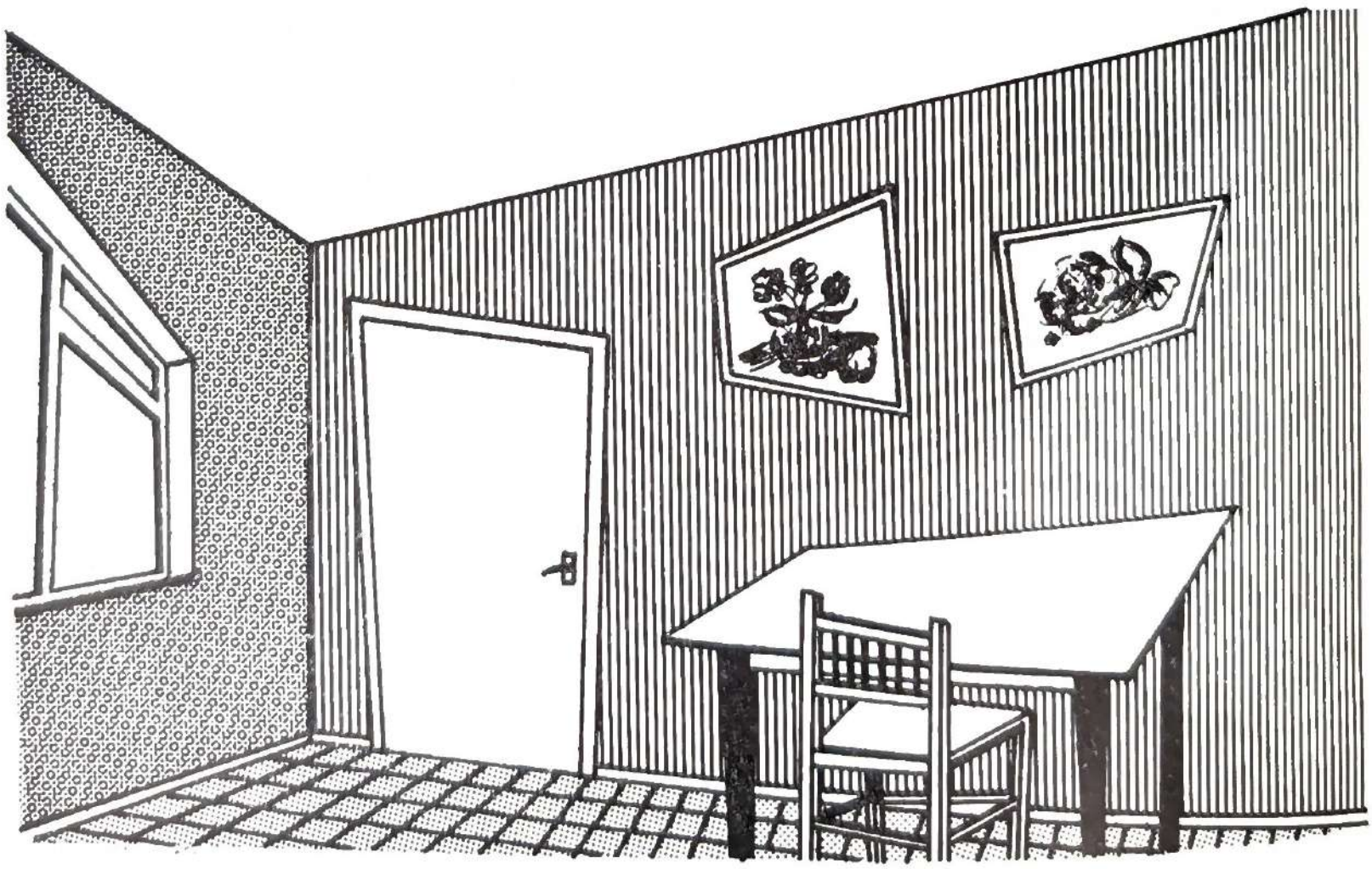
CHAPTER 2

WHY ARE THINGS THE SHAPES THEY ARE?

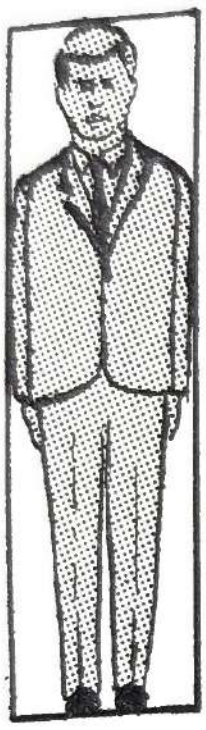
In this chapter we will try to discover why some of the things around us are a certain shape.

Everywhere you will see rectangles. This page is rectangular. Perhaps the shape of the top of your table is a rectangle. The door, the windows, walls and the picture frames have rectangular shapes—and there will be many other rectangles to notice. How many different ones can you see? Make a list. Perhaps you will be surprised to discover so many things of this shape.

Why are rectangles so common? Well, let us see how a room would look without them. In the drawing the shapes are not

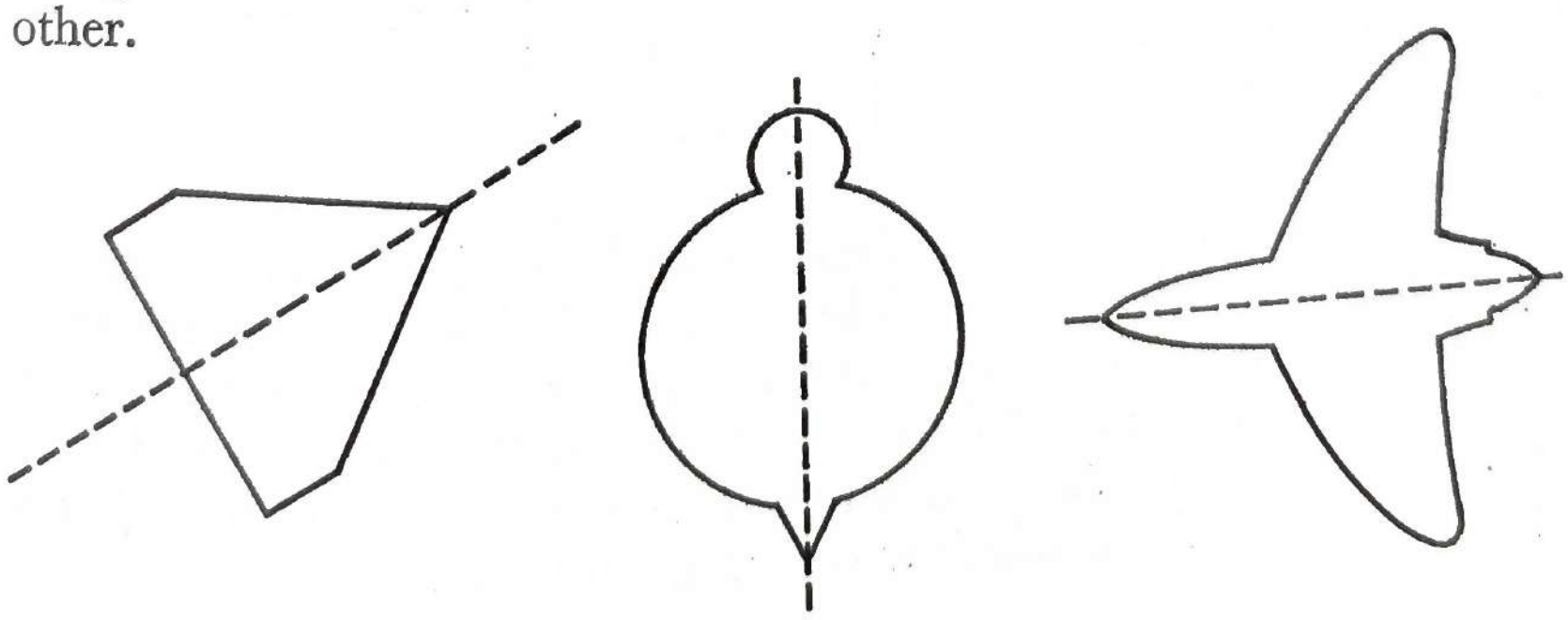


rectangles because they have no right-angles. Everything seems wrong. One reason for this is that human beings walk in an upright position. We hold our bodies vertically. What is more, a drawing of a human fits well inside a rectangle.

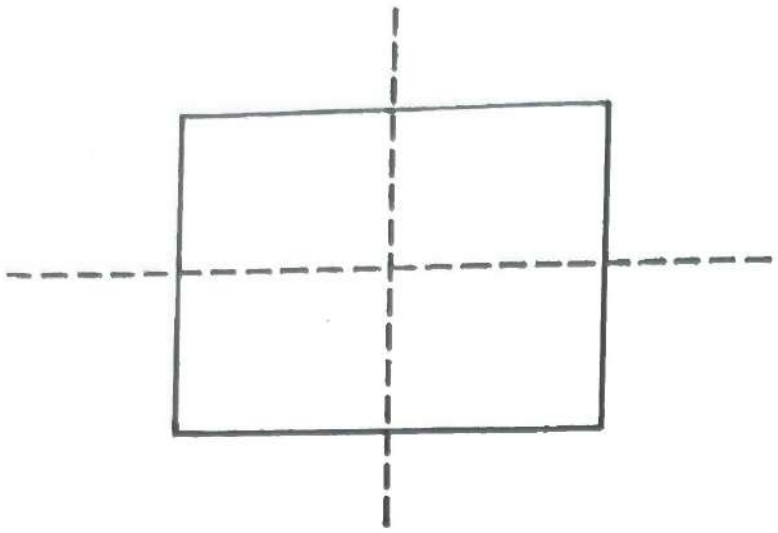


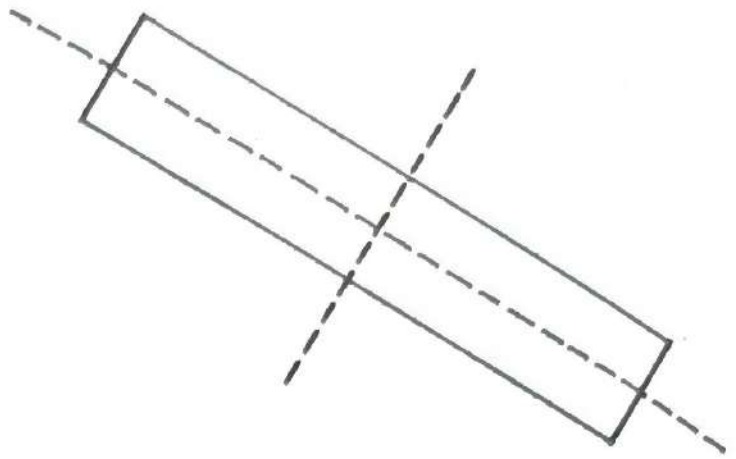
As we fit into rectangular shapes easily when we stand upright it, would be silly to make doors any other shape.

A rectangle is a pleasing shape. Some shapes are pleasing, and some are not. Shapes that are pleasing are usually *symmetrical*. A symmetrical shape is one that is balanced along one or more imaginary lines. For example, all the shapes drawn below are symmetrical along the dotted lines. If the shapes were folded along these lines, then the two halves would fit exactly on to each other.

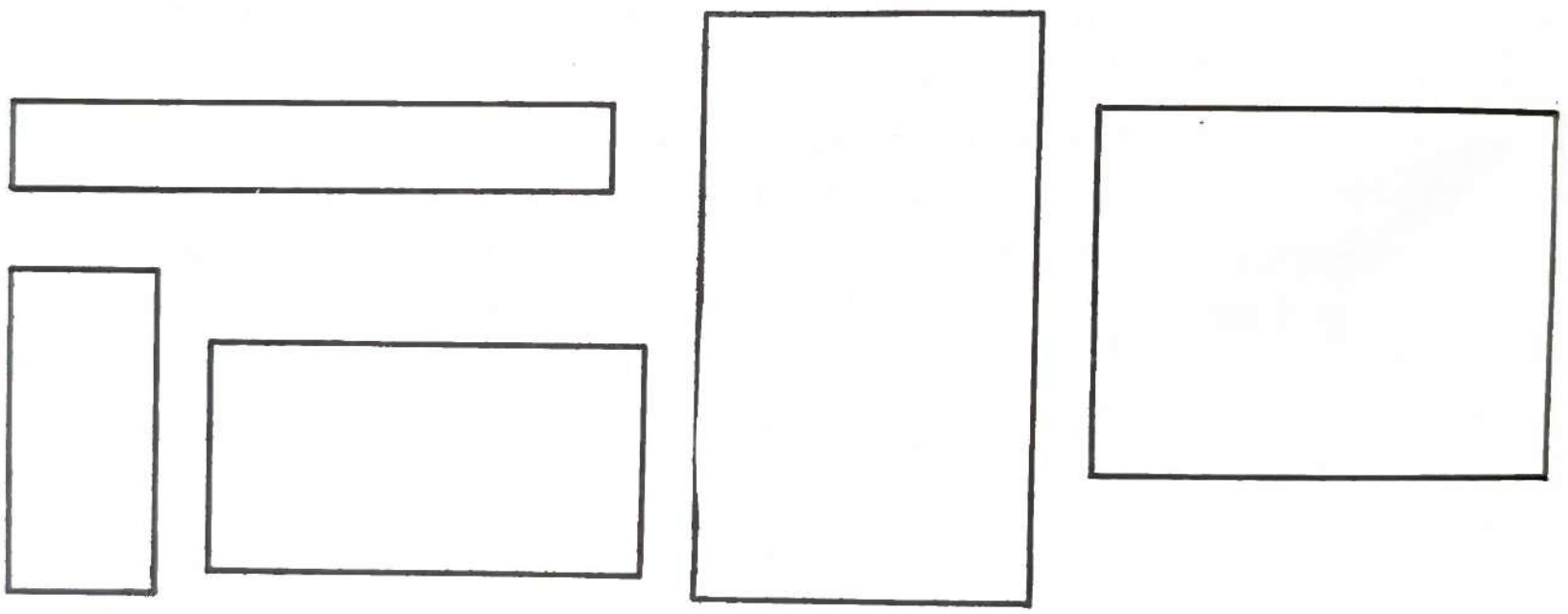


Each dotted line is called an axis of symmetry. Rectangles have more than one axis of symmetry:

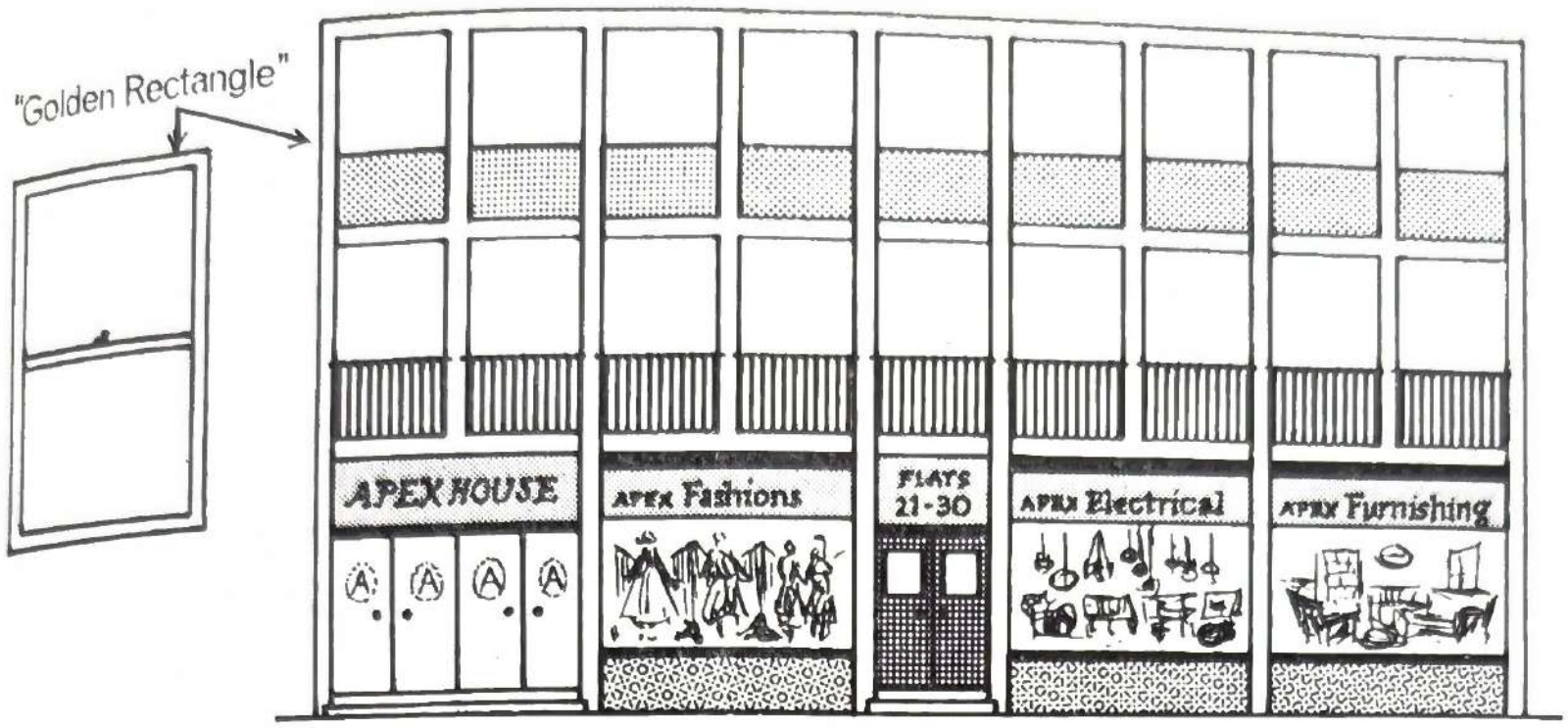




Some rectangles are more pleasing than others. Which of the following rectangles do you like best?

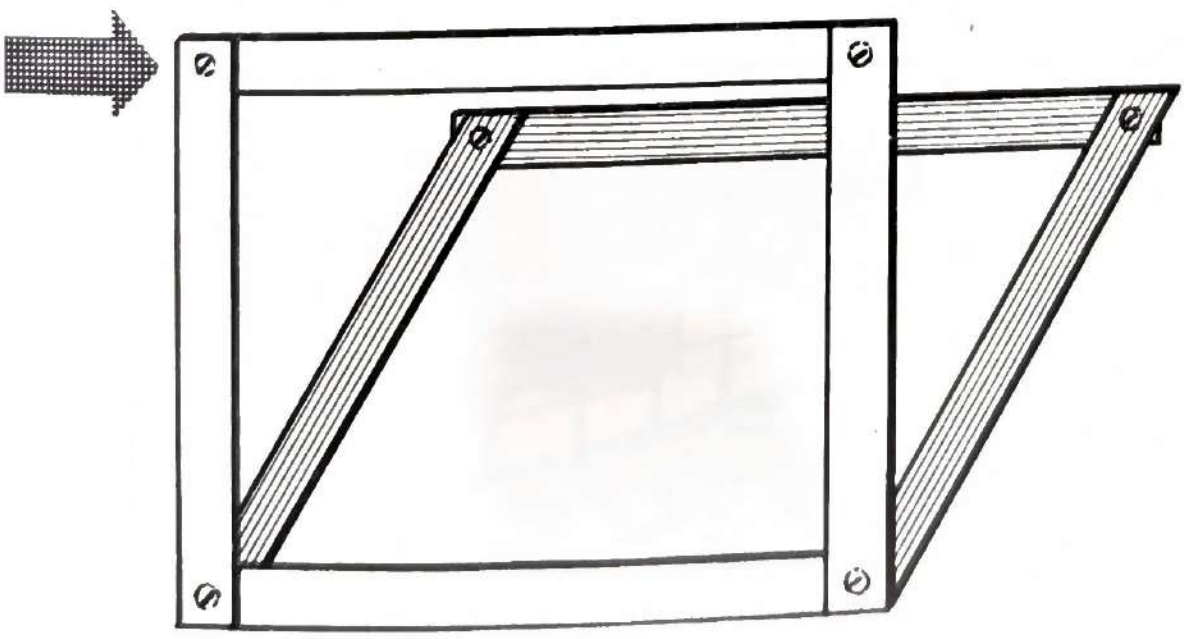


One kind of rectangle is often used. The sides have a special relationship to each other. The shorter side is a little more than 0·6 (that is, $\frac{6}{10}$th.) the length of the longer side. Rectangles with sides in this ratio are often called 'Golden Rectangles', and were well known to the Ancient Greeks. As you can see, it makes a pleasing shape for a window or for a building.



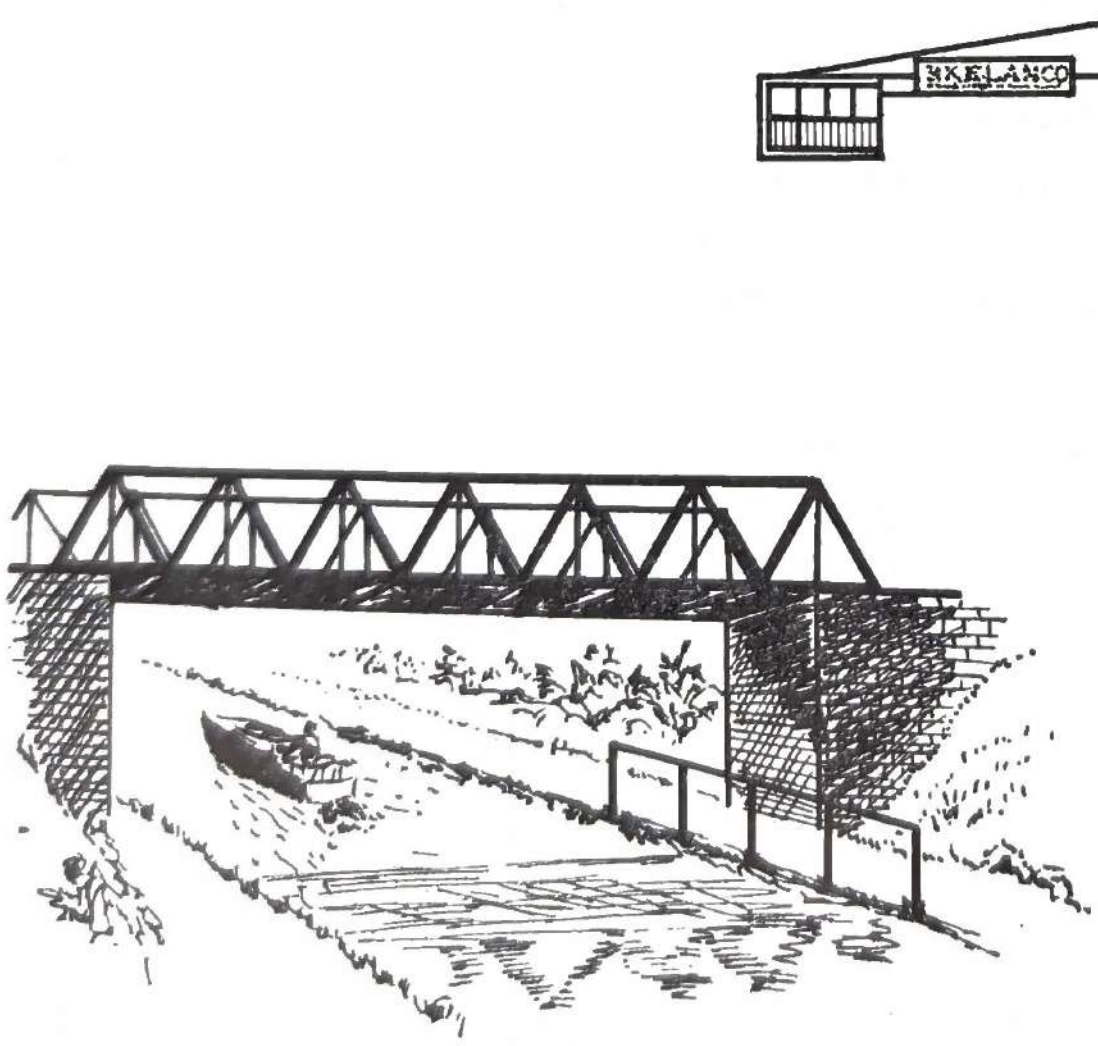
Rectangles are not only pleasing, they are shapes that fit together very well. Many things that have to fit together are made in the shape of squares. You can find square tiles for the floor, or for the walls and ceilings, and square paving slabs for out of doors.

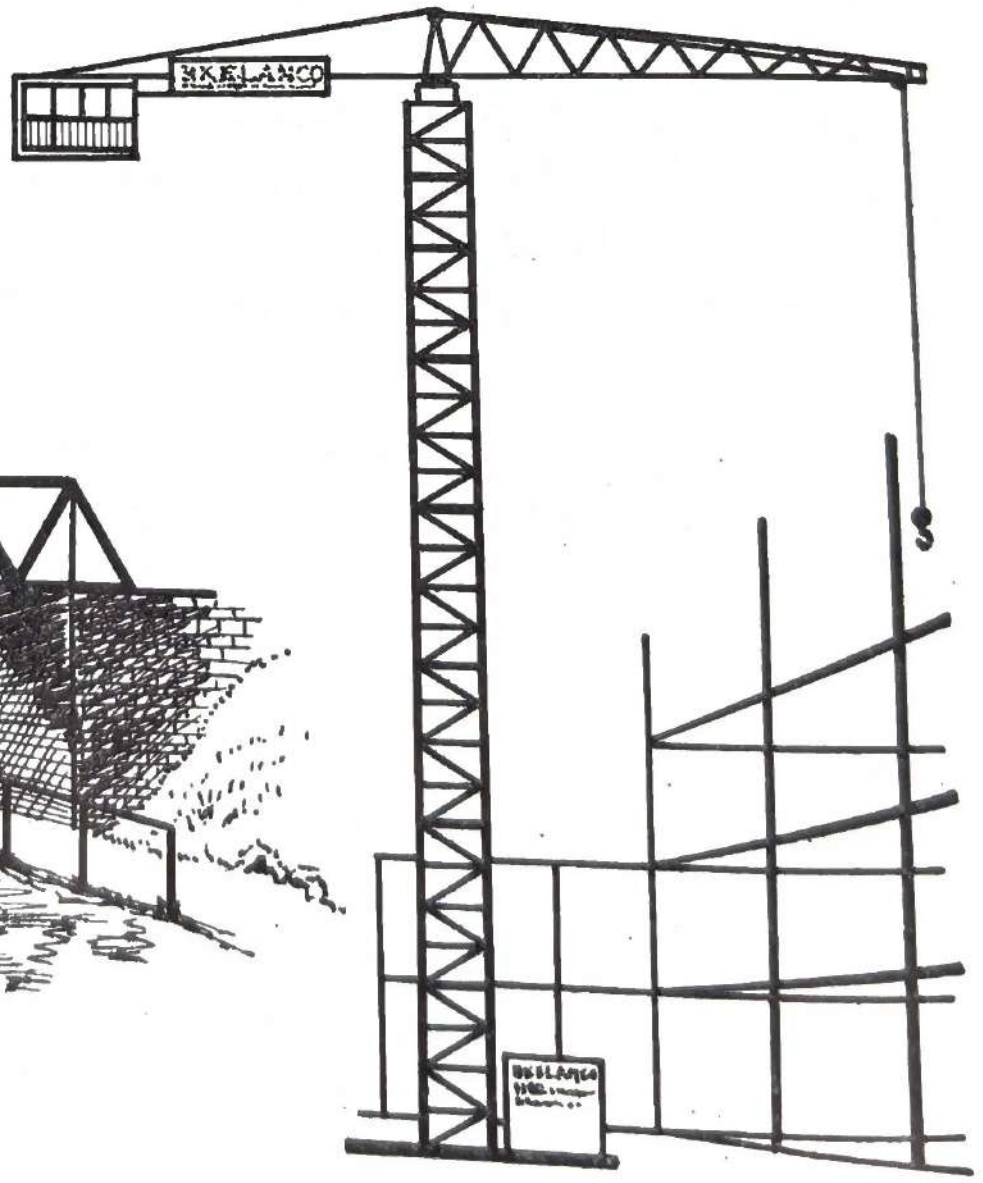
If you take some pieces of wood and make them into a rectangle by screwing the pieces together, the shape is not a strong one. No matter how well you tighten the screws, the shape can be pushed so that the corners are no longer right-angles:



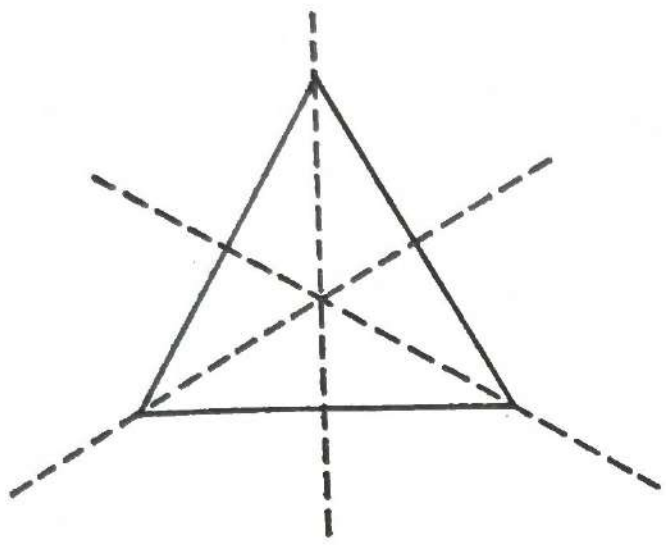
Rectangles are not suitable for things that must be very rigid. It would be useless to build a tall tower, such as a pylon for electricity cables, with pieces of steel bolted together in the form of rectangles. The tower would not be strong enough.

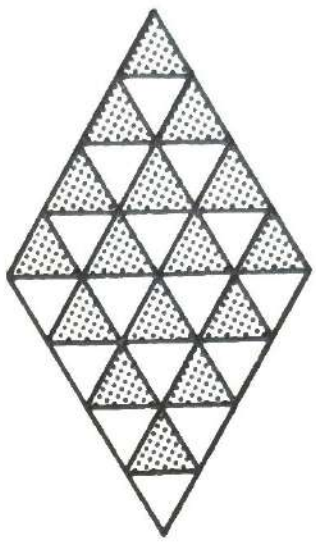
Rectangles have 4 sides, but, strange to say, a shape with just 3 sides is stronger. If you take three pieces of wood and screw them together to make a triangle, this shape cannot be altered, even if the screws are not very tight. You can try this for yourself. Take 4 burnt matches and 4 pins. Lay the 4 matches in the form of a rectangle and join the corners by pushing the pins carefully through. This shape can now be changed by taking one corner in each hand and pushing or pulling gently. Now take three matches, make them into a triangle and join the corners. This shape will not change. It is a very strong shape that is used in building bridges and other things, as you can see from the drawings below.



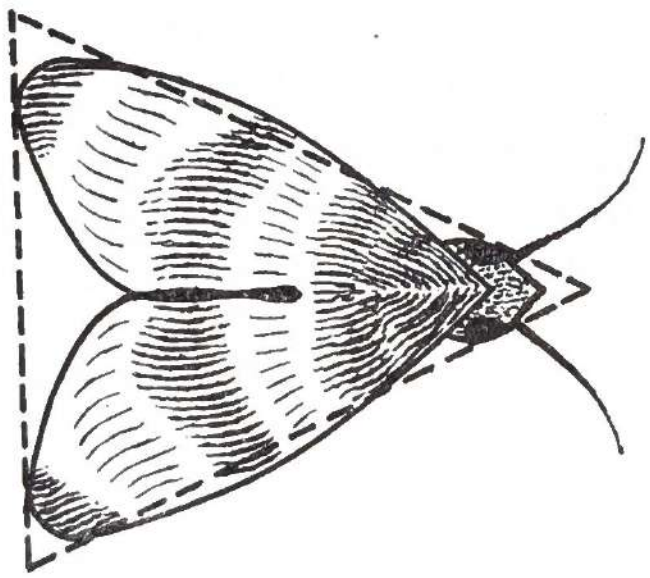


A triangle with 3 sides of equal length is called an *equilateral* triangle. Equilateral triangles are pleasing to look at because they have three axes of symmetry (see page 21). They will also fit together without leaving spaces. If you take a number of equilateral triangles of the same size in two different colours, many interesting patterns can be made.



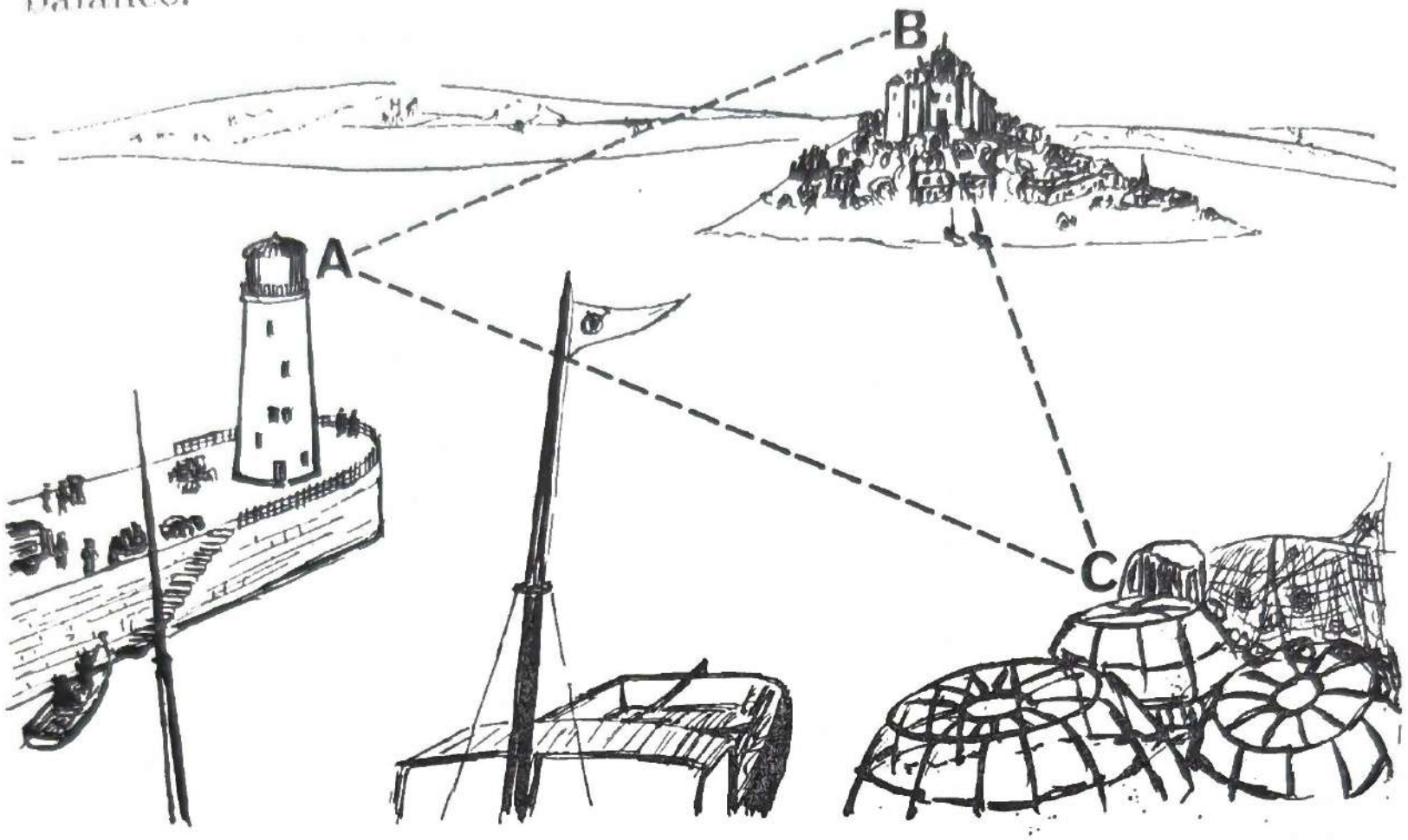


Many things are triangular. Some of the latest jet 'planes have wings that form a triangle, but it is not only man-made things that have this form. Some butterflies and moths have an almost triangular shape when seen from above with their wings folded.

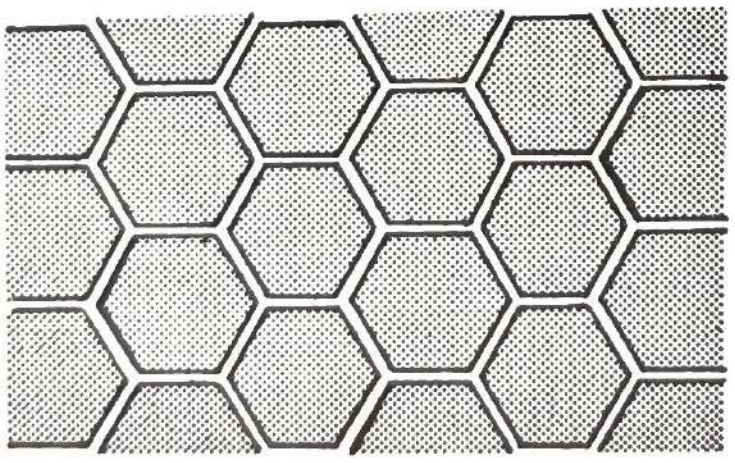


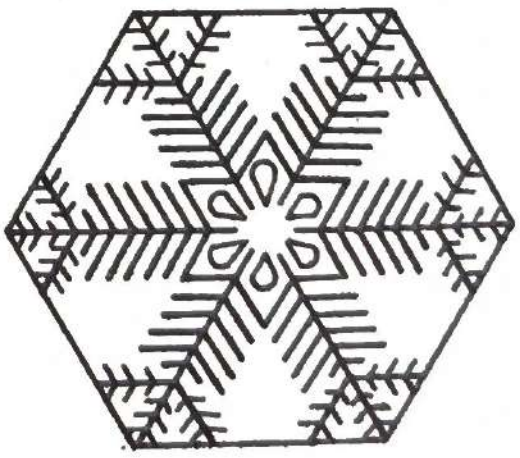
You can often see a triangle in a painting. This does not mean that the artist paints actual triangles, but that he places important things on points in the picture so that they form a triangle. Look carefully at pictures and see if you can discover this.

In the drawing below, the important parts of the picture, labelled A, B and C, form a triangle, and this helps to give the picture balance.

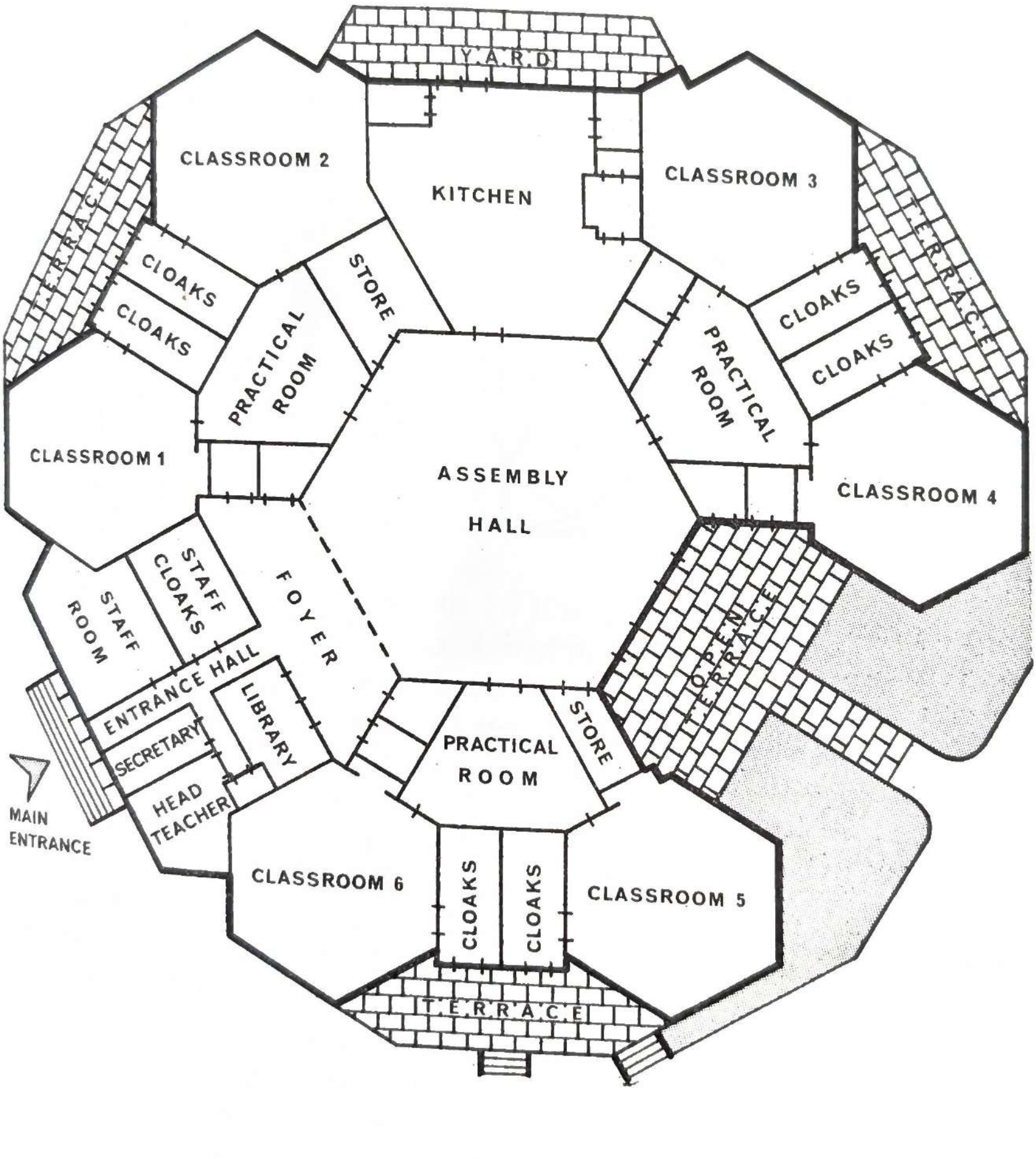


One other polygon seen in the natural world is the hexagon. The honey bee builds its honeycomb with cells of this shape. The cells fit together well, they are strong and hold a great deal of honey. In fact, the hexagon is the best possible shape for this purpose. Snowflakes, too, will fit inside hexagons. The tiny crystals of ice are formed into shapes with six arms. The points of these arms, if joined, would make a hexagon.

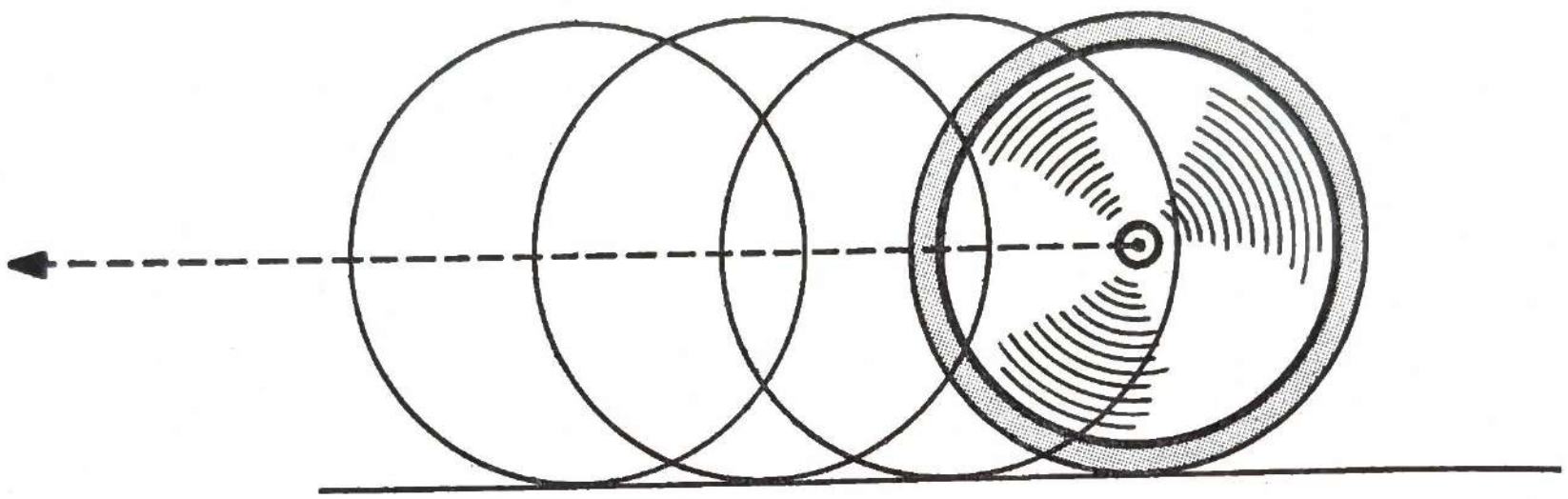




In Leicestershire, schools have been built with hexagonal rooms so that no space is wasted. Even the hall is hexagonal, and the hexagonal classrooms and other spaces fit round it. See if you can design a school or a house like this.



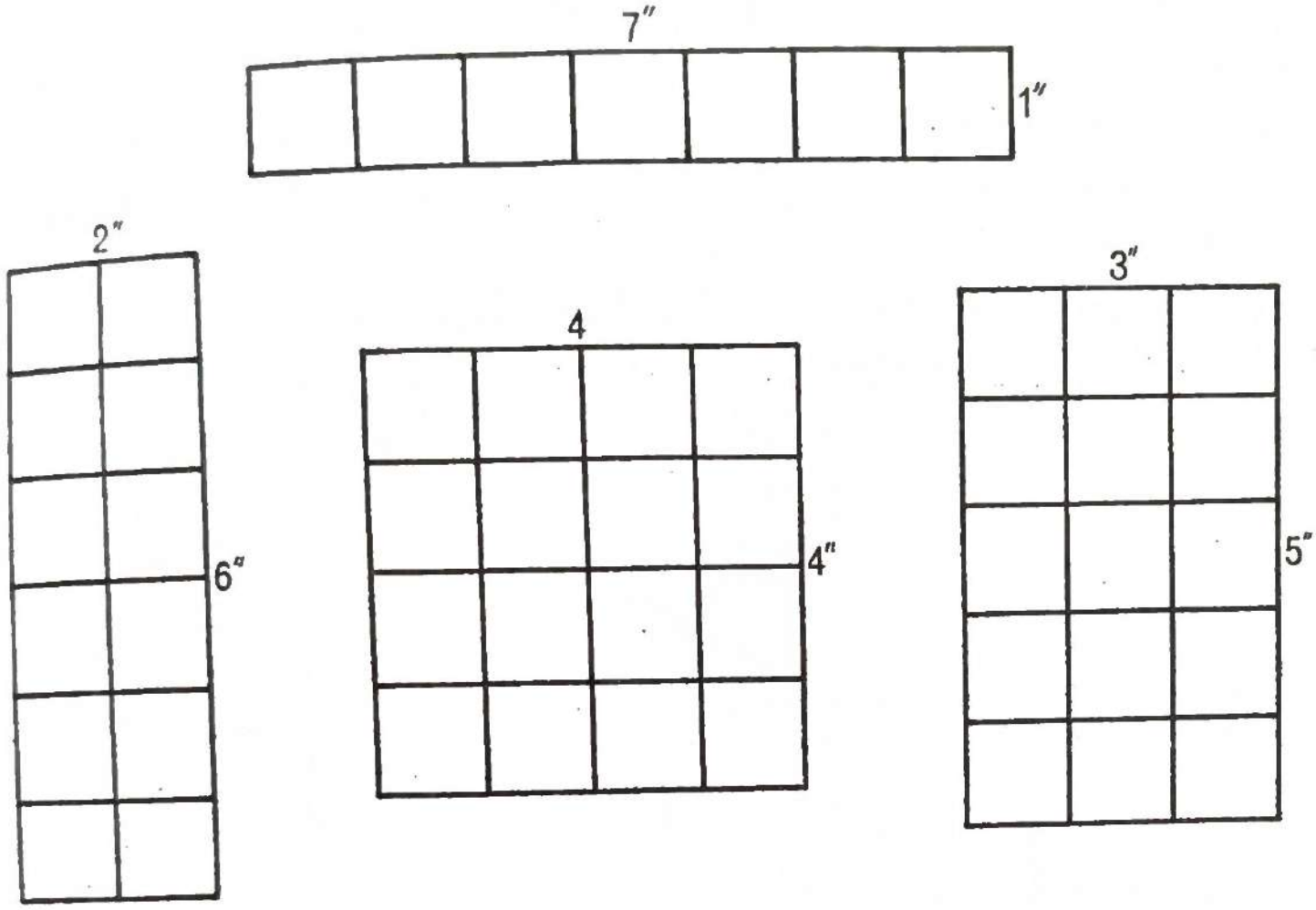
One of the most important things ever invented is the wheel. Wheels are used for a great many purposes. Bicycles, cars, and trains would be useless without wheels. Have you ever thought why a wheel is circular? Well, the answer is a mathematical one. You will remember that we learnt (on page 15) that there is just one point inside every circle which is equally distant from every point on the circumference. Because of this, if a wheel is circular, the axle can be placed at this centre. Then, when the wheel rolls along a flat surface, the axle remains at the same height and moves in a straight line.



You can try this for yourself by making a wheel from thin card and pushing a pin or matchstick carefully through the centre. If you then hold the ends of the match between finger and thumb and roll the wheel along the top of a table, keeping it upright, you will see and feel your fingers moving in a straight line parallel to the table top.

Wheels are not only used for helping to move things smoothly. Gears are really wheels with teeth cut around the circumference. The teeth of gears interlock so that one wheel turns, or is driven by, the other. Some gear wheels are very small. You can see small gear wheels in your wrist watch. It is worthwhile examining the gears through a magnifying glass, but be very careful not to allow them to become dirty. What is the biggest gear wheel that you can find? Keep a sharp lookout. You may see some large gear wheels on cranes or other large machinery.

Another useful and interesting fact about circles is to do with area. Given 16 inches of string, what is the largest area that you could enclose with this length? Suppose you made rectangles like the ones shown below:

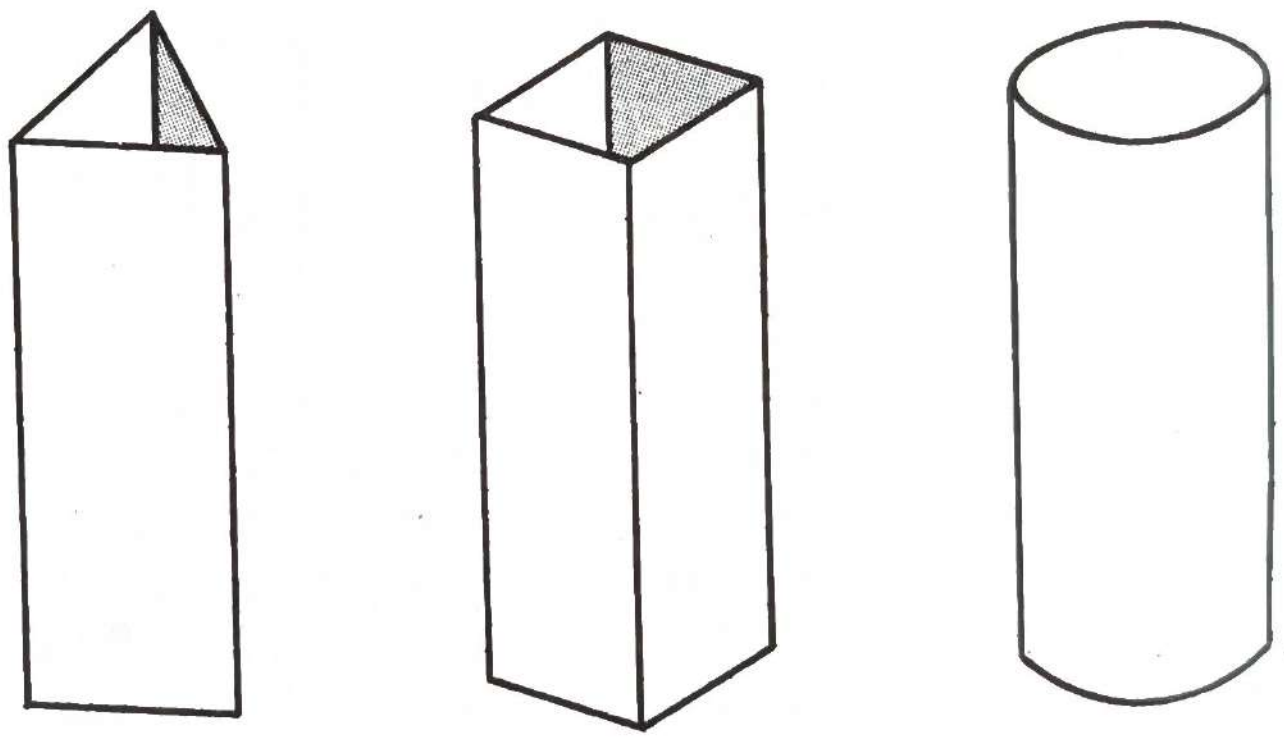


The distance round the outside of each shape is the same. It is 16 inches. But the area of each shape varies. As you can see, of the shapes made the square has the largest area, but the largest *possible* area would be a circle! This is a useful fact. If you want to cut the largest hole in a piece of wood, with the least possible work, then the hole would be circular. Windows in the sides of the hull of a liner are called 'portholes'. They are circular. Holes cut in the sides of a ship cannot be too large. The circular window lets in the most light because it has the biggest possible area for any given length of cut.

Another reason for some things being circular is that they wear evenly. There are no corners to rub off. Can you think of some things that are circular for this reason?

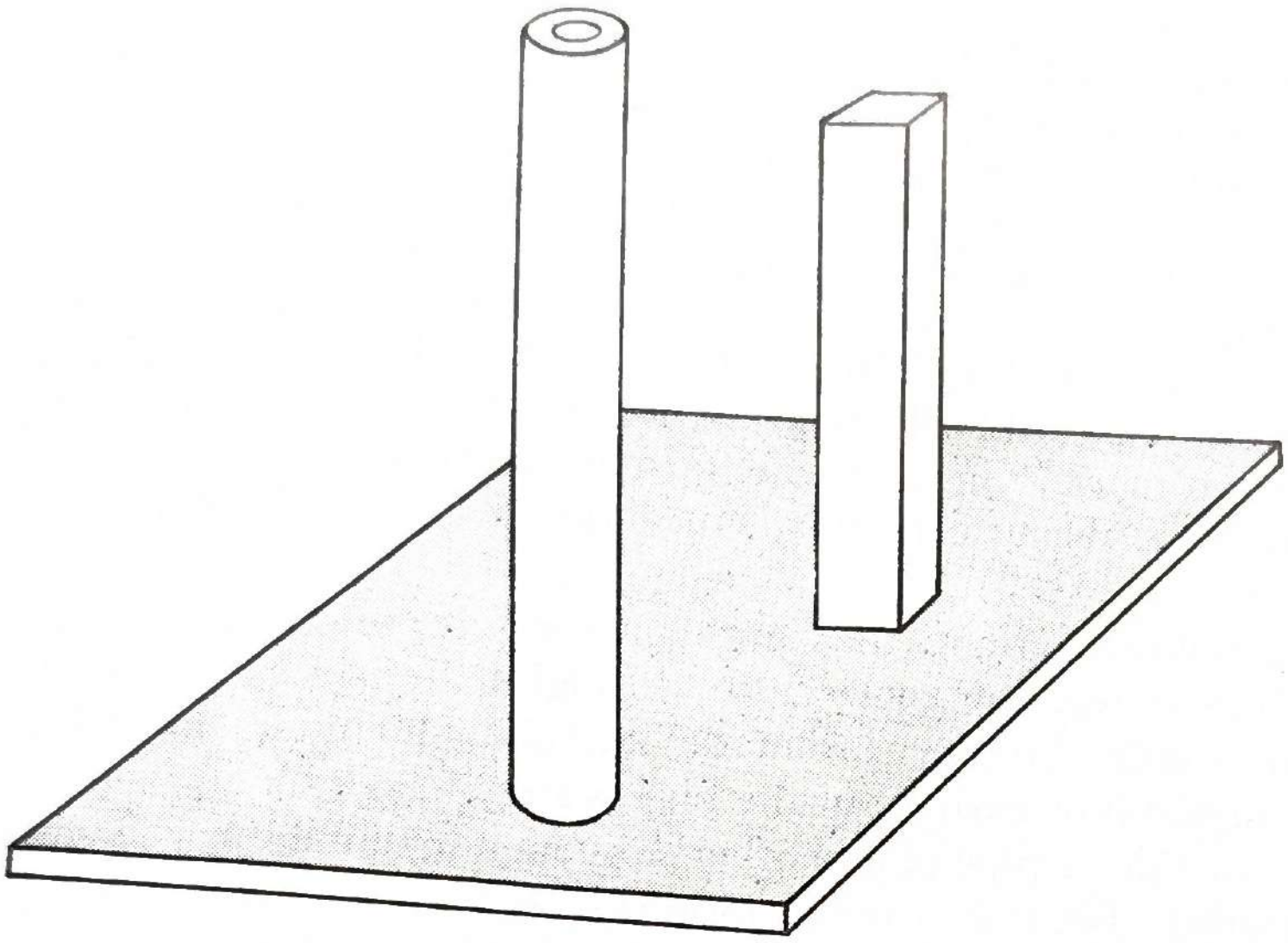
As you know, cylinders have circular bases. Many things, both natural and man-made are cylindrical. We can discover the reasons for this by trying some simple experiments. First you will need three pieces of thin card. These pieces should be rectangular and should be the same size. Length and width are not important, but pieces of card about as big as the pages of this book are excellent.

The first experiment shows that the biggest container that you can make with one sheet of card will be cylindrical. Take three pieces of card and shape them carefully so that each is the same height but one has a triangular base, the second a square base, and the third a circular base. Join the edges of the card with sellotape. The containers will look like this:



The next thing to do is to pour dry sand into each container until it is full. Although each container is made of the same amount of material, you will find that the cylinder will hold more sand than the other shapes. You can test this by weighing the sand. This is one reason why many tins to hold food or liquids are cylindrical.

Cylindrical shapes are very strong, even if the cylinders are narrow. Experiment with some plasticine. Take two balls of plasticine of equal size. With one, build a *solid* tower with flat sides. With the other, build a cylindrical tower that is hollow. Both towers should be the same width. The hollow tower will be higher than the solid one:



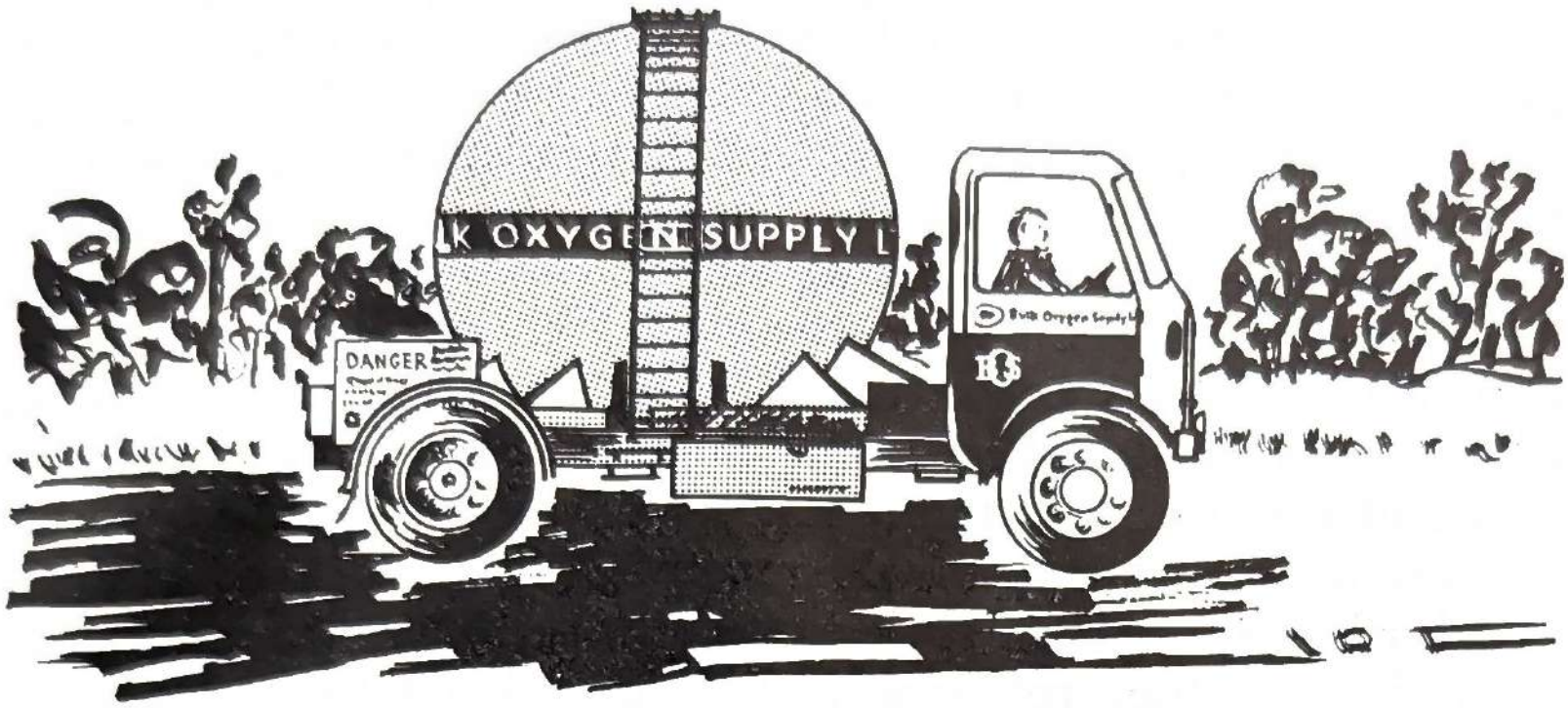
You can test the strength of the towers in several ways. One way is to pile pennies on top and see which tower will support more of them. Another way is to hold the base of each tower firmly, and then push the top sideways with increasing pressure. This test is not very exact, but you will be able to feel which tower bends more easily. If you have moulded the plasticine together properly and kept the wall the same thickness from bottom to top, the cylinder should not bend easily even though it is taller.

The fact that hollow cylinders do not bend easily suggests that tall, narrow things should be made in this way. Collect tall plants such as reeds or grasses and examine the stalks. You will find that they are really very slender hollow cylinders! In the middle of London, the Post Office has built a very tall tower. If you visit it you will find that the centre is a strong, hollow cylinder containing the lifts and other things. The outside walls and windows form a cylinder also. Several towns in various parts of the world have tall towers like this. All of them are hollow cylinders! The columns and pillars built many years ago are this shape also. The columns of the ancient Greek temples were cylinders, although not all of them were hollow, and some became thinner at the top. One well-known hollow cylinder is the Monument which stands near London Bridge in London and was built to remind us of the place where the Great Fire of London started in the year A.D. 1666. It is not only tall buildings that are made in this way. The longest and strongest bones in the human body are also like hollow cylinders. Because of this, our large bones are lighter and less likely to break.

A hollow cylinder has another very important use. Liquids, such as water, flow more easily through cylindrical tubes than any other shape. So pipes are made like this. Of course, not only does the liquid flow easily, but the pipe is strong and allows more water to flow than a pipe of any other shape made of the same amount of material. Because a metal pipe is so strong, it is very difficult to bend! Special equipment has to be used to do this. If a plumber comes to your house ask him about this.

Why do so many things have the shape of a sphere? The sphere is a very important shape because it contains the greatest volume for the least surface. For example, if you wanted to make a box to contain 1 cubic foot of sand, the container requiring the least amount of material would be spherical. Of course, spheres are very difficult to make, so it would not be worth the trouble for sand! The second very important thing about the sphere is

that there is the same pressure over every part of the surface. Look at this drawing of a lorry carrying liquid oxygen:



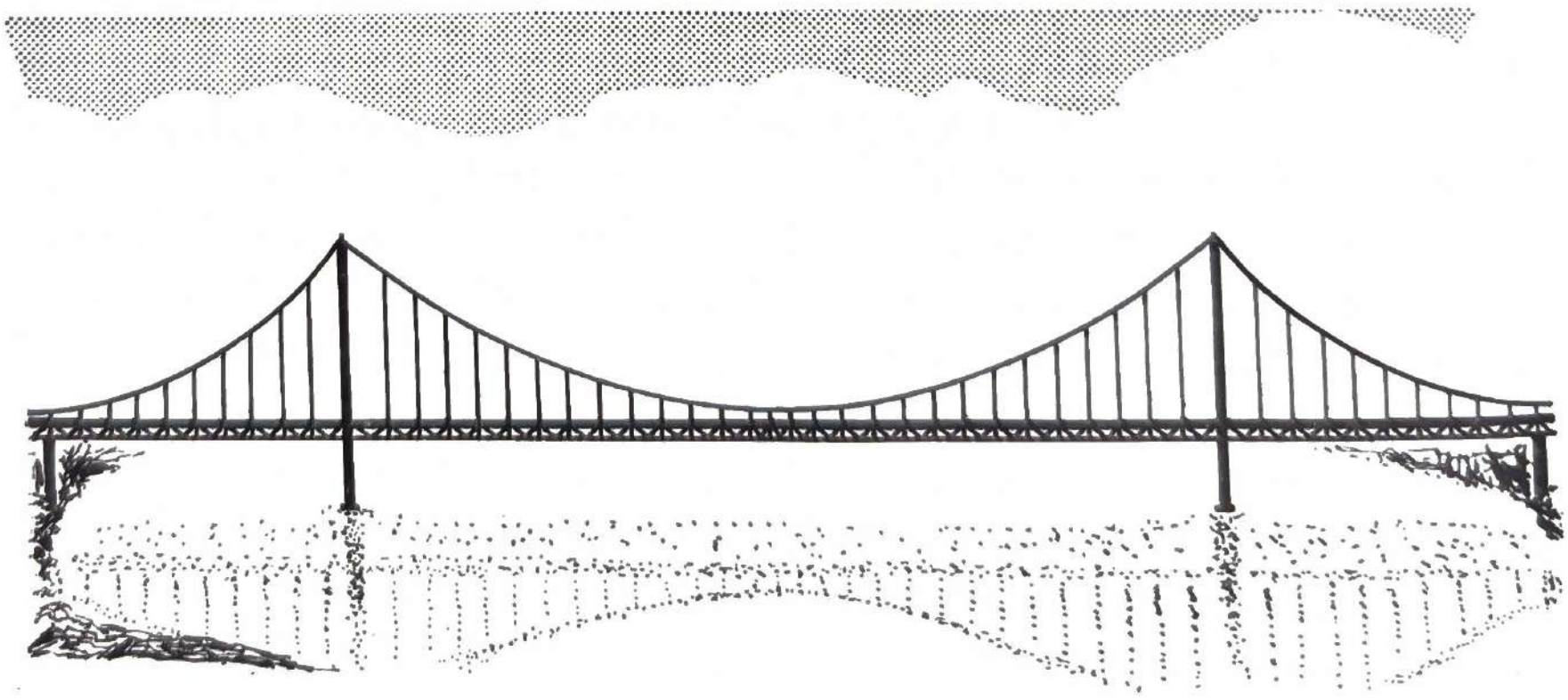
Liquid oxygen is made by compressing the gas, oxygen. The liquid tries to change back into gas so the pressure inside the container is very great. As you can see from the drawing, the container is spherical. Because of this, no part of the surface has to stand greater pressure than any other part—and, what is more, the spherical shape holds more liquid than any other made from the same amount of steel.

Spheres are very strong shapes and they form naturally if liquids or gases have to stand up to pressure from all around. Take a straw and blow very slowly and gently into a tall glass of water—or any other liquid. The air forms bubbles that are almost spherical. Stir up a bottle of 'fizzy' lemonade (with the stopper removed) and watch the tiny bubbles of the gas, carbon dioxide, rise to the surface. These bubbles are spherical. When a jet of water leaves a pipe and meets the pressure of the air, the stream breaks up into drops after a short distance. These drops, as they fall to the ground, are spheres that have been pushed out of shape. It is not easy to see this with your eyes because everything happens so quickly, but try to do so with a dripping water tap. One day

you might see a slow-motion film of this. The *best* way to see spheres form is to blow soap bubbles! One interesting use of the fact that liquids form into spheres when dropped in air was in the making of lead shot. In London, near the Royal Festival Hall by the River Thames, there is an old 'shot tower' where lead shot was once made. Molten lead was dropped from top to bottom and formed into small spheres which became solid as they fell.

Spheres are symmetrical no matter how you look at them. This is why some people consider the sphere as the perfect shape. Balls will roll in a straight line if the surface is smooth, unless the ball is unevenly balanced. The bearings for wheels of bicycles, cars and machinery of many kinds are *ball* bearings because the spheres wear evenly and roll smoothly. Spheres move so easily that birds that lay their eggs on the ground do not lay eggs of this shape in case they roll away! Try to find out more about this from other books or by looking at collections of birds' eggs.

In chapter 1 we mentioned the mathematical shape called the parabola. This shape is very useful, but you may not notice it around you. One of the biggest parabolas in Great Britain can be seen on the new Firth of Forth road bridge in Scotland. The huge cables that hold the road are this shape. There are more parabolas to be seen than you think! Electric fires often have



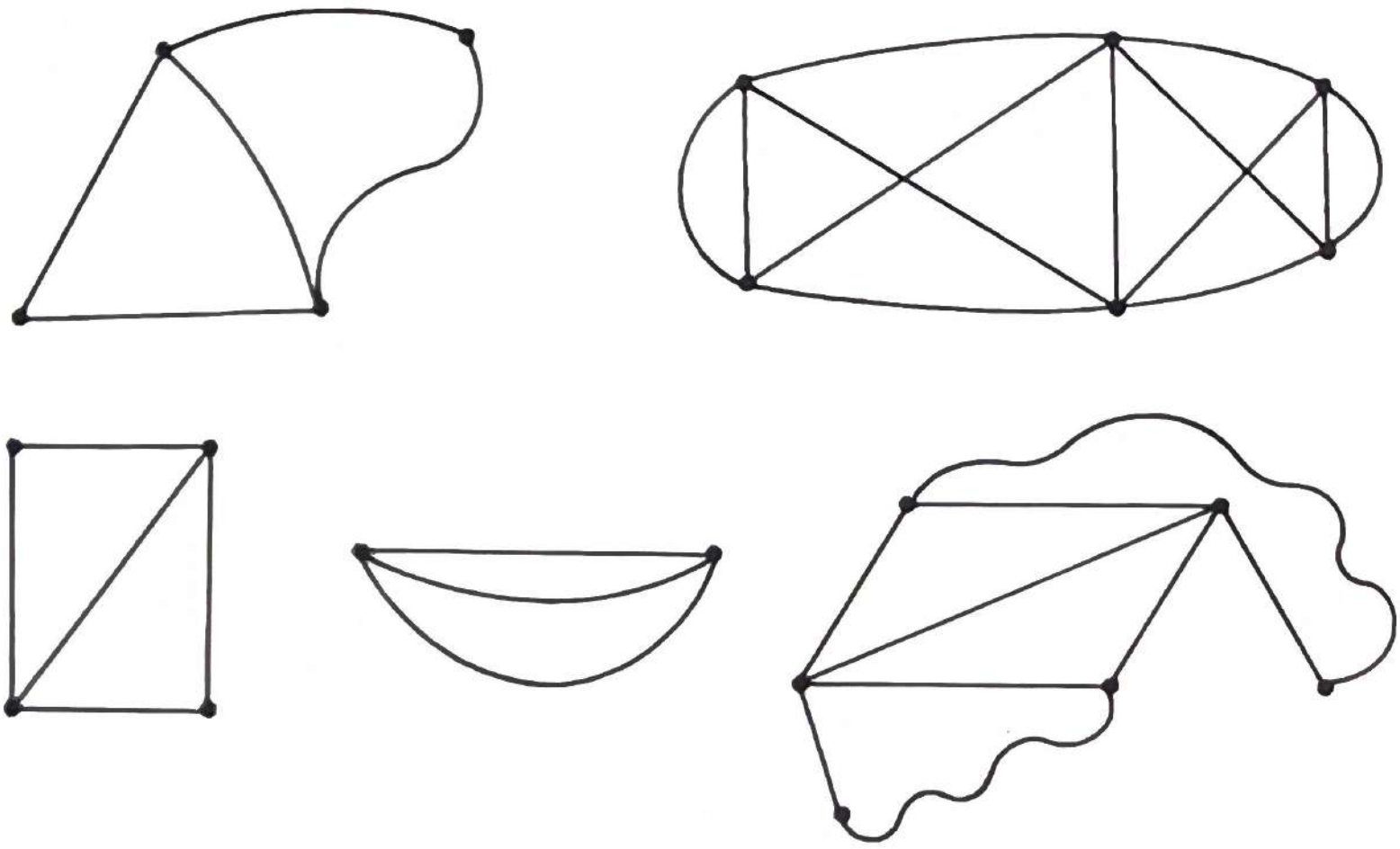
parabolic reflectors and so do headlamps of cars and torches. The reason for this is that the parabola reflects the heat or light in rays that are parallel and equally strong. How many more parabolas can you discover?

This chapter has told you something about the reasons for the mathematical shapes of some things. There is much more to discover and learn. As you work and play examine the shapes of other things and try to find out more about them.

CHAPTER 3

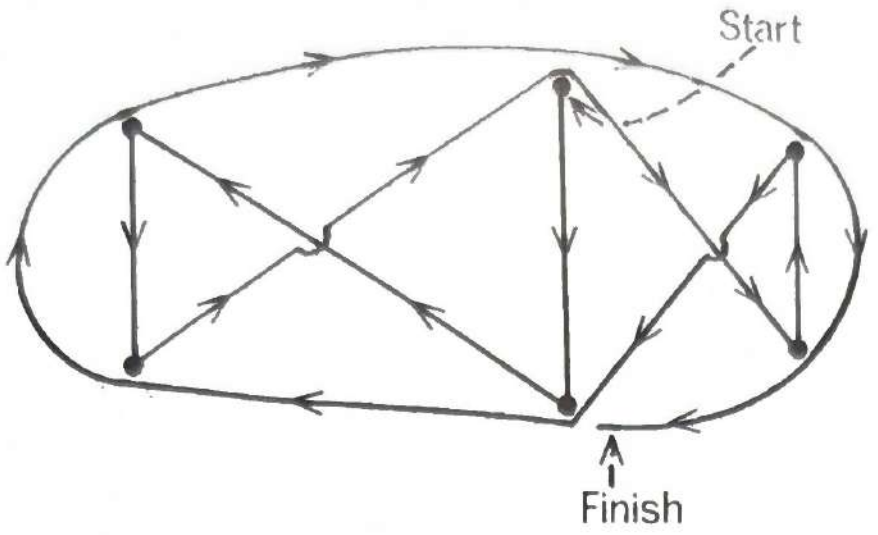
SOME NEW WAYS OF LOOKING AT SHAPES

THE strange shapes drawn below are formed from lines meeting at points. Such shapes are called 'networks'. The lines are called 'arcs' and the points are called 'vertices'. All the shapes are alike in a certain way. Can you say how they are alike? The clue that will help you is to pay attention to the vertices.



Well, as you can see, each shape has just 2 vertices where an *odd number* of arcs meet. Any network like this can be drawn with one continuous line and without going over any part of it more

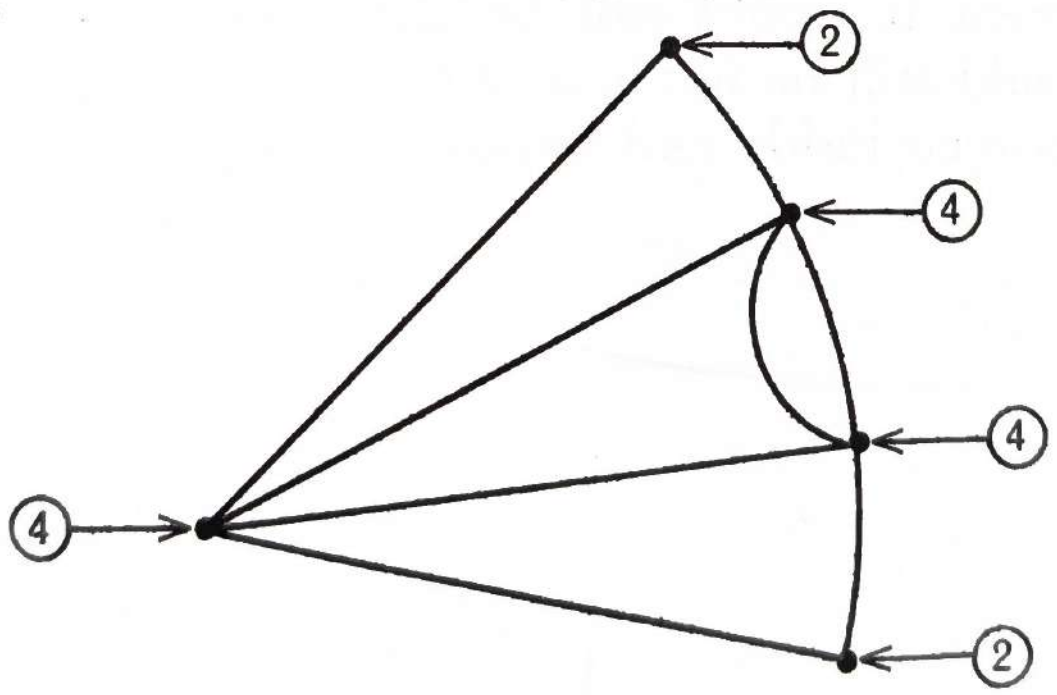
than once. Here is one of the drawings again showing how this may be done:



Can you see how to draw the other shapes with one continuous line?

Invent a few shapes of your own. Be sure that each has only 2 vertices where an odd number of arcs meet.

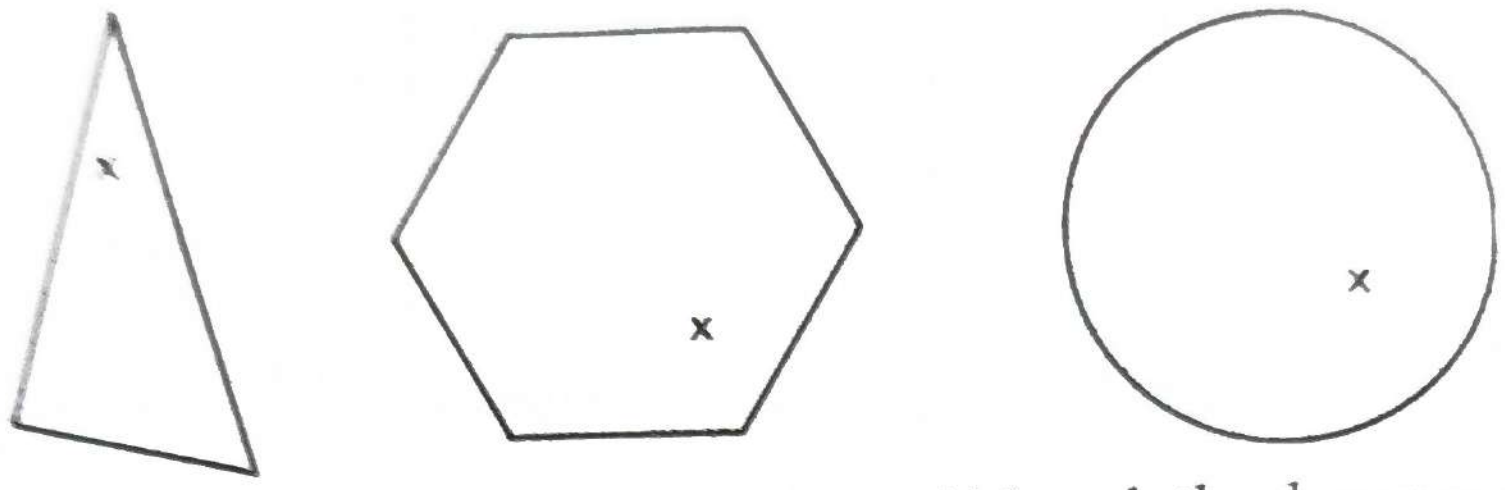
When you have done this, draw some networks with no vertices where an odd number of arcs meet. The drawing below shows a network of this kind. The arrows pointing to the vertices show the number of arcs:



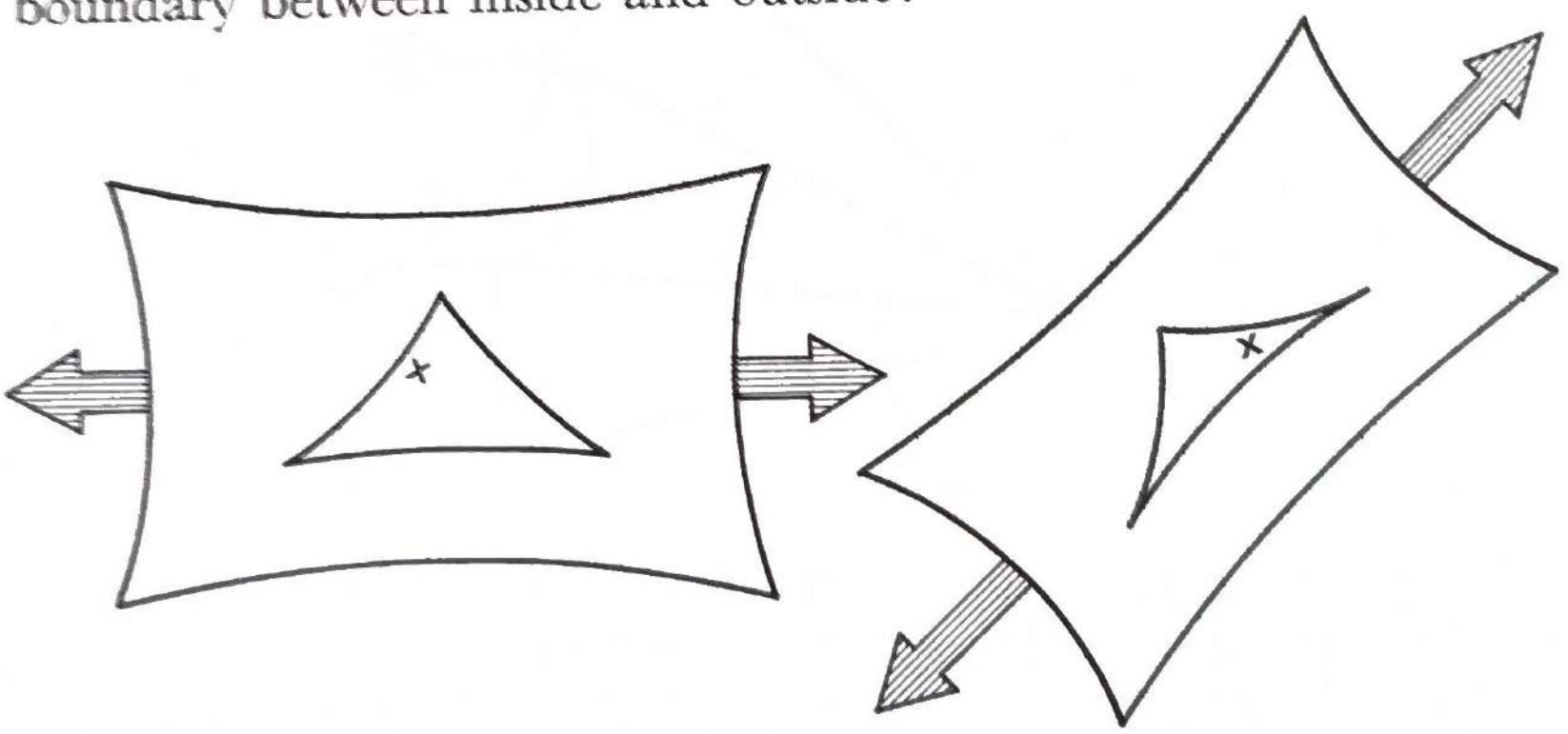
All the vertices have an even number of arcs. Can networks of this kind be drawn with one continuous line?

What else can you discover about networks?

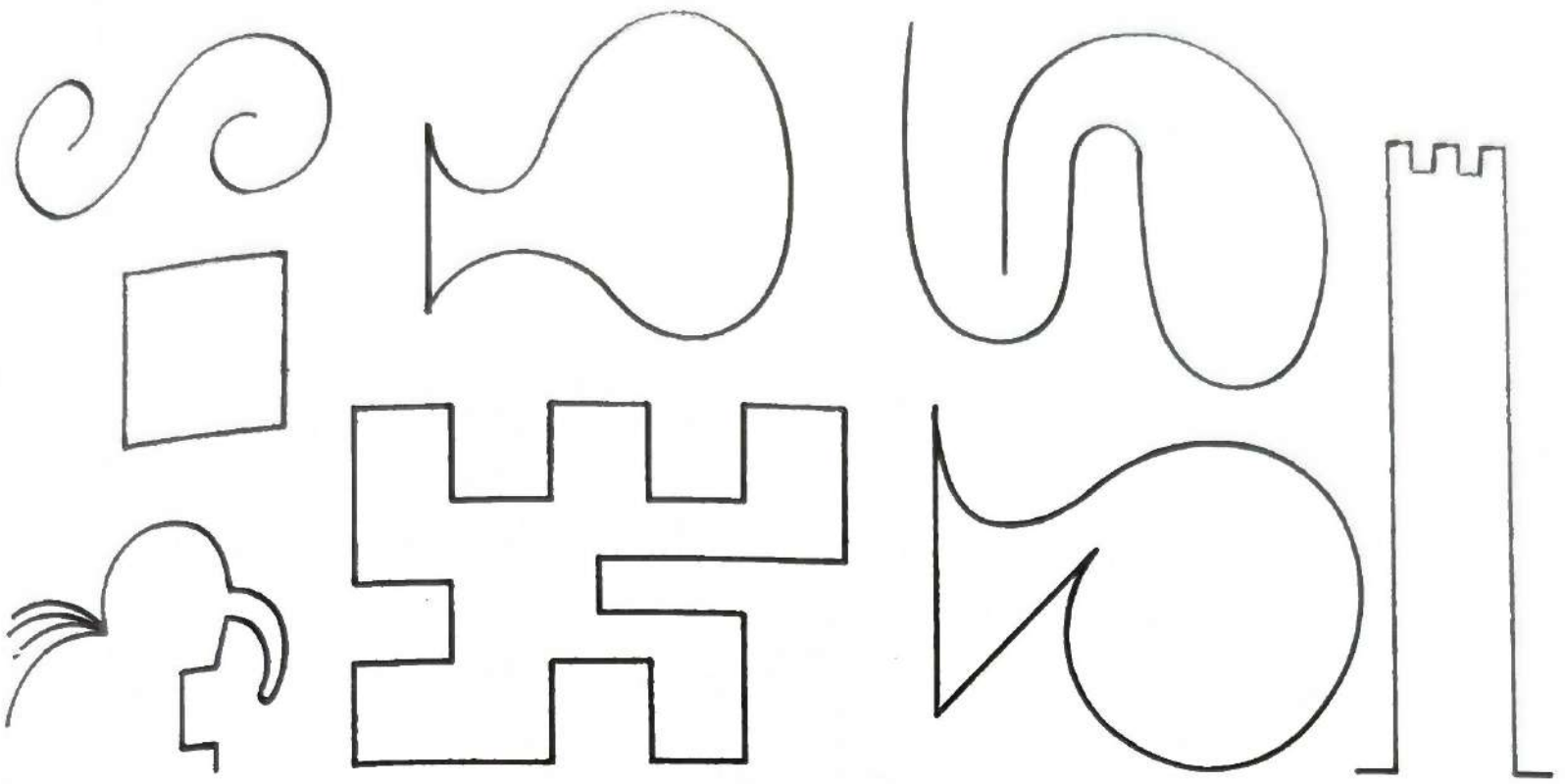
Perhaps you find it strange that shapes which look so different are alike in certain ways! Here is another example. The drawings show a triangle, a hexagon and a circle. Each has a small cross marked, but the shapes do not seem to have anything else in common:



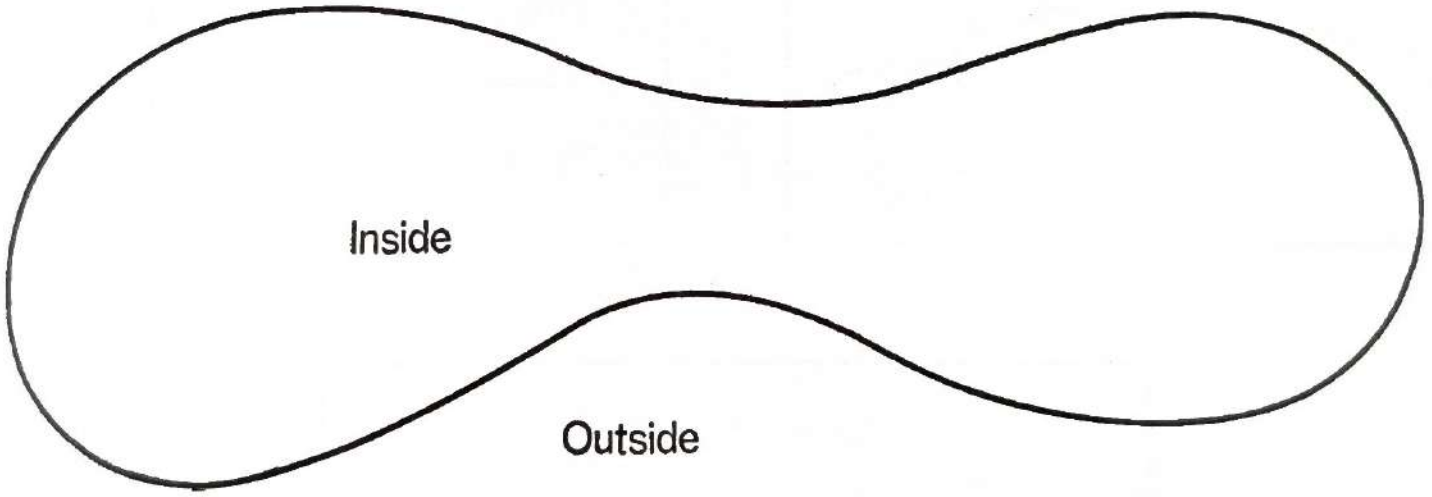
Imagine an insect at each of the points. Although the shapes vary, in each case, the insect would have to cross the boundary in order to get outside the shape. Any one of the shapes drawn on a piece of material that could be stretched, such as a sheet of thin rubber, would divide the surface into two parts—an inside and an outside. No matter how the rubber sheet was stretched so that the shape seemed different it would still be the same in this own way. The cross would still be inside and there would be a continuous boundary between inside and outside:



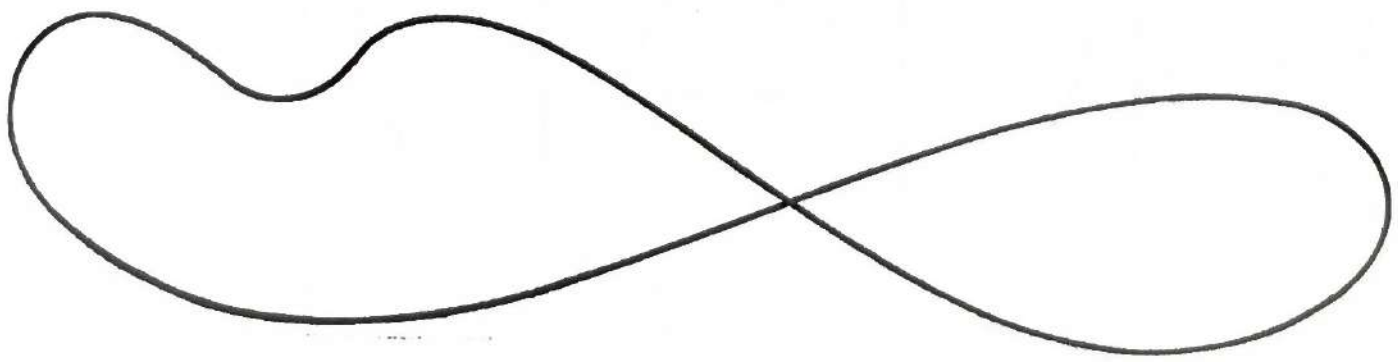
Looked at in this way, which of the following shapes are alike?



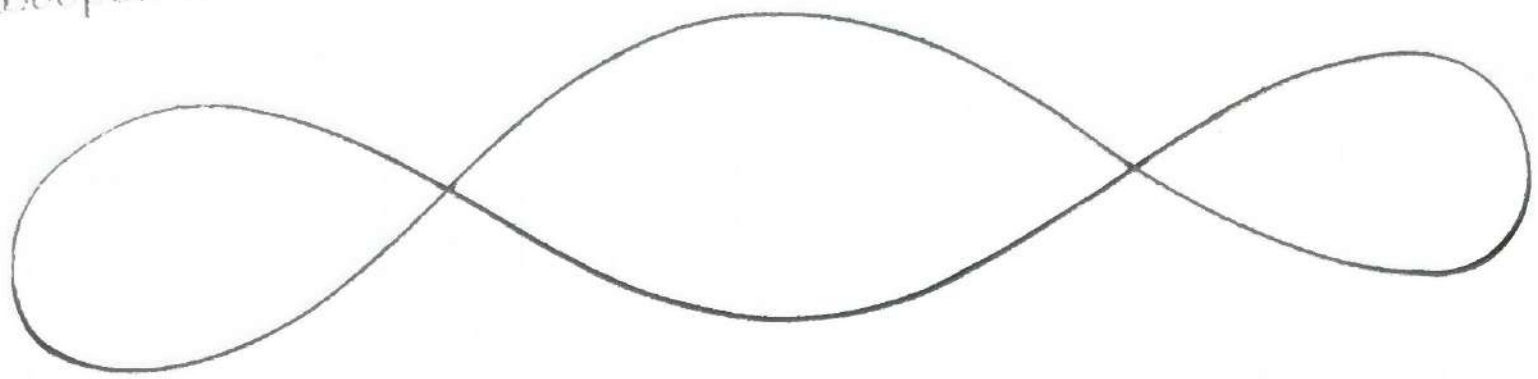
Of course, shapes can have more than one region inside. A piece of string looped like this has 1 region inside and 1 region outside:



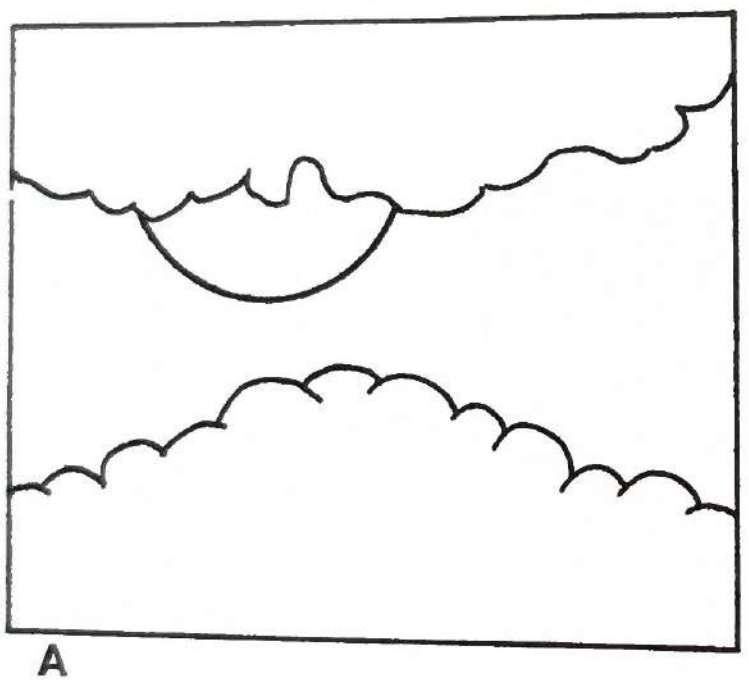
Looped like this it has 2 regions inside and 1 region outside:



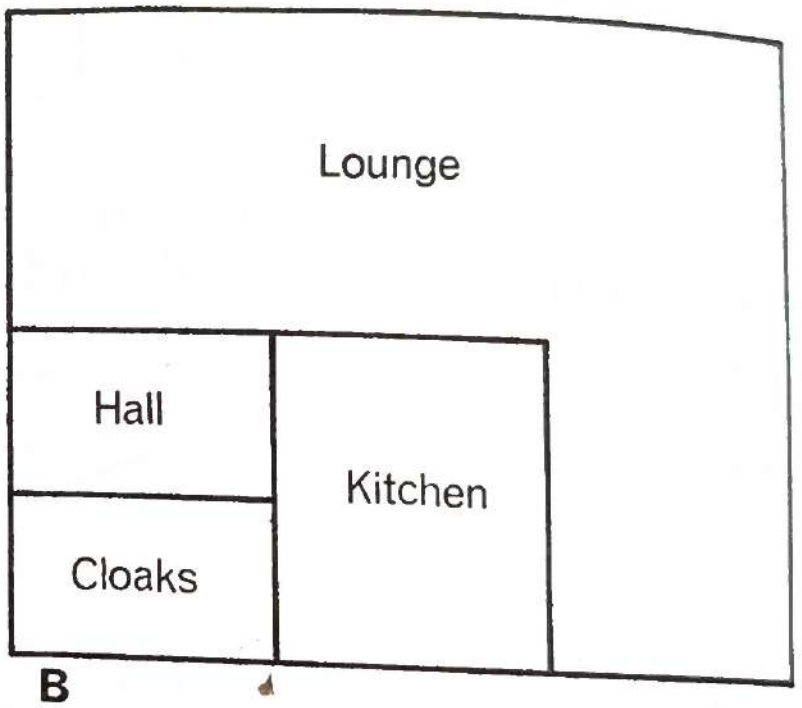
Looped like this it has 3 regions inside and 1 region outside:



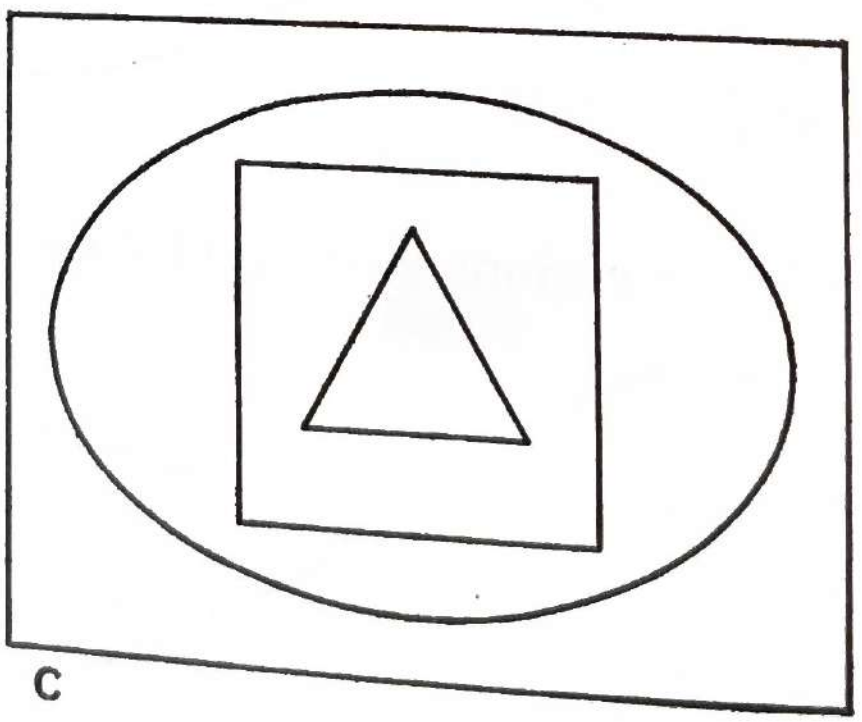
Looked at in this way, the following drawings are similar. Drawing A shows the moon in a cloudy sky. Drawing B is the plan for a house. Drawing C shows some mathematical shapes inside each other. Each drawing divides the space into 4 regions:



A

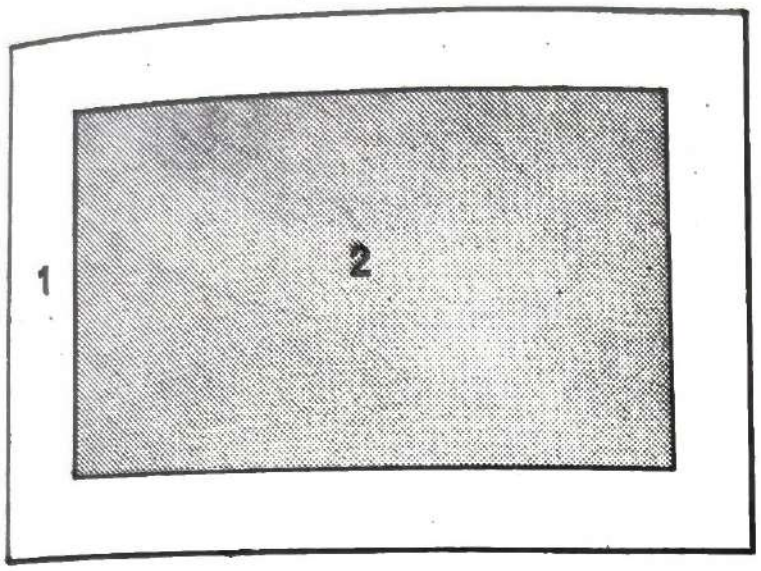


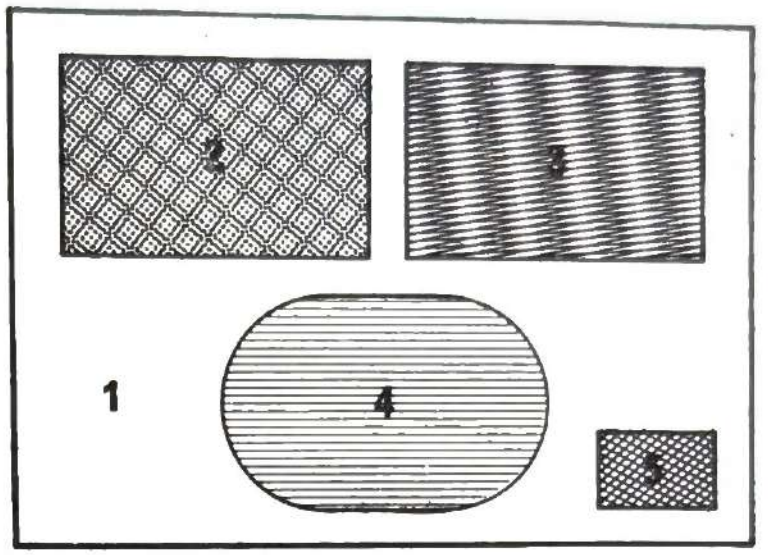
B



C

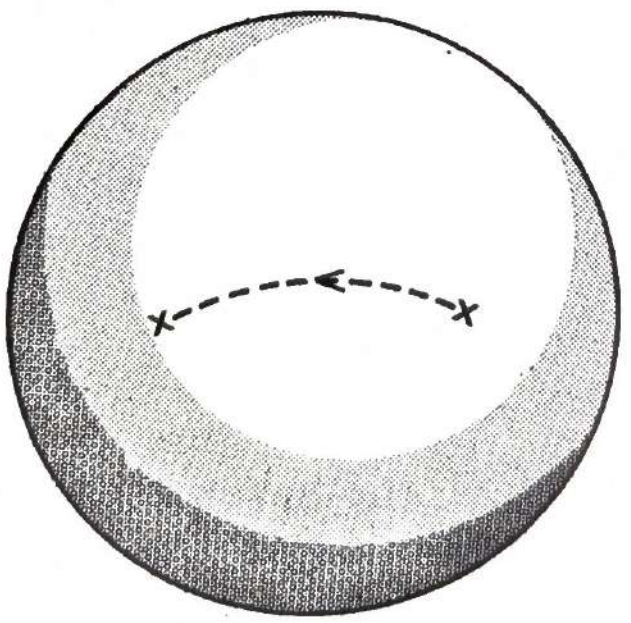
You can see the regions clearly if you colour them. Make some drawings yourself that have more than 4 regions. Then look around you at the walls and floors to see if you can find surfaces that are like this. For example, a carpet on the floor divides it into just 2 regions—but four small rugs or mats divide it into 5 regions:



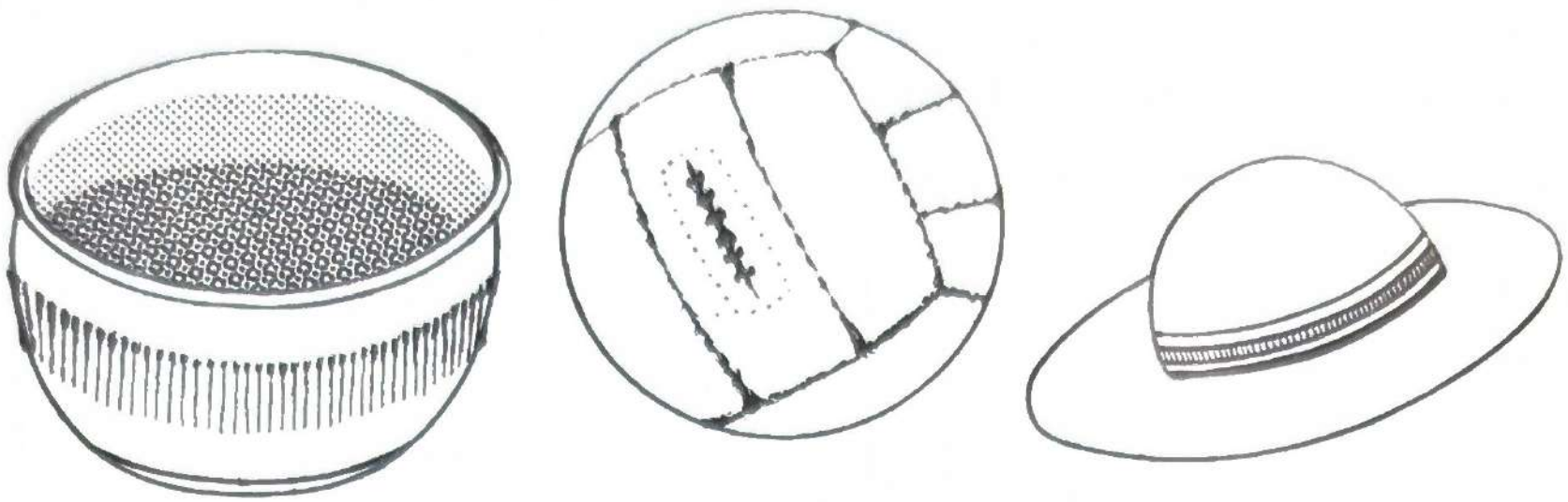


So far we have talked about regions of flat surfaces. What happens when we consider solid objects? The best way to learn about these is to have some plasticine or clay to experiment with.

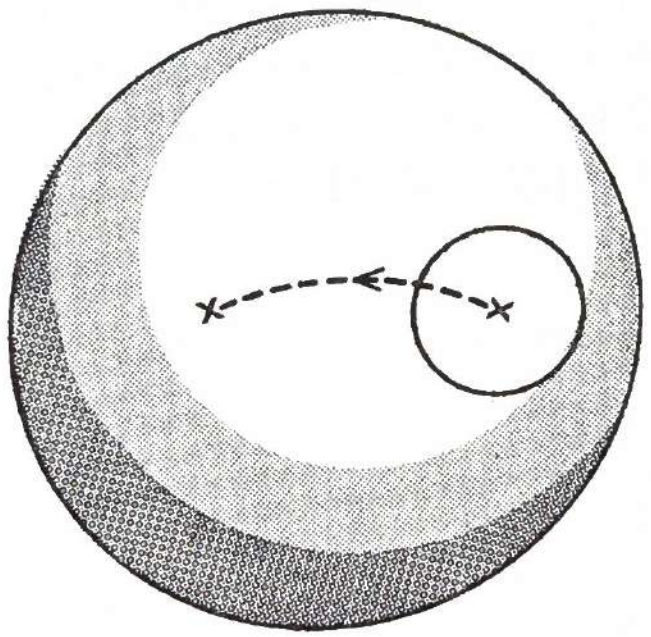
The first thing to do is to make a sphere of clay. The outside surface of the sphere is continuous. If two points are marked anywhere on this surface, an insect could crawl from one to the other without crossing any boundary:



Look at the drawings below. You can see a bowl, a football and a hat.



At first glance these seem to be quite different from the sphere of plasticine, but, if you make them from the sphere, you will see that each has a continuous surface. If a closed curve is drawn on a sphere, it divides the surface into two regions and the curve forms a boundary. An insect at any point *inside* the closed curve cannot get to the *outside* without crossing this boundary:

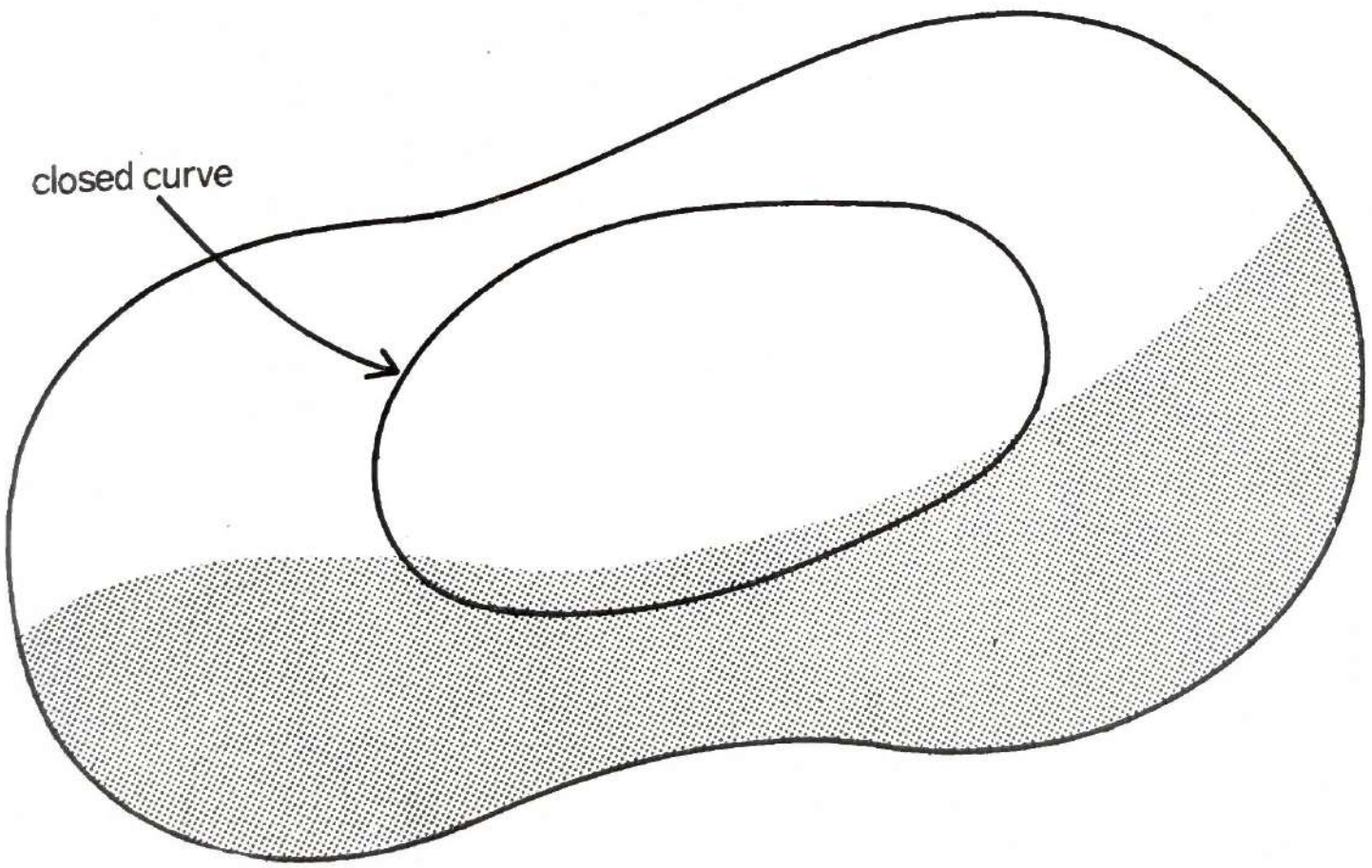


The same is true of the bowl, the hat and the football. No matter where you draw a closed curve, or how small or large it is, the

curve forms a boundary and breaks up the surface into two regions. Try drawing some closed curves on the shapes you have made and you will discover that this is true.

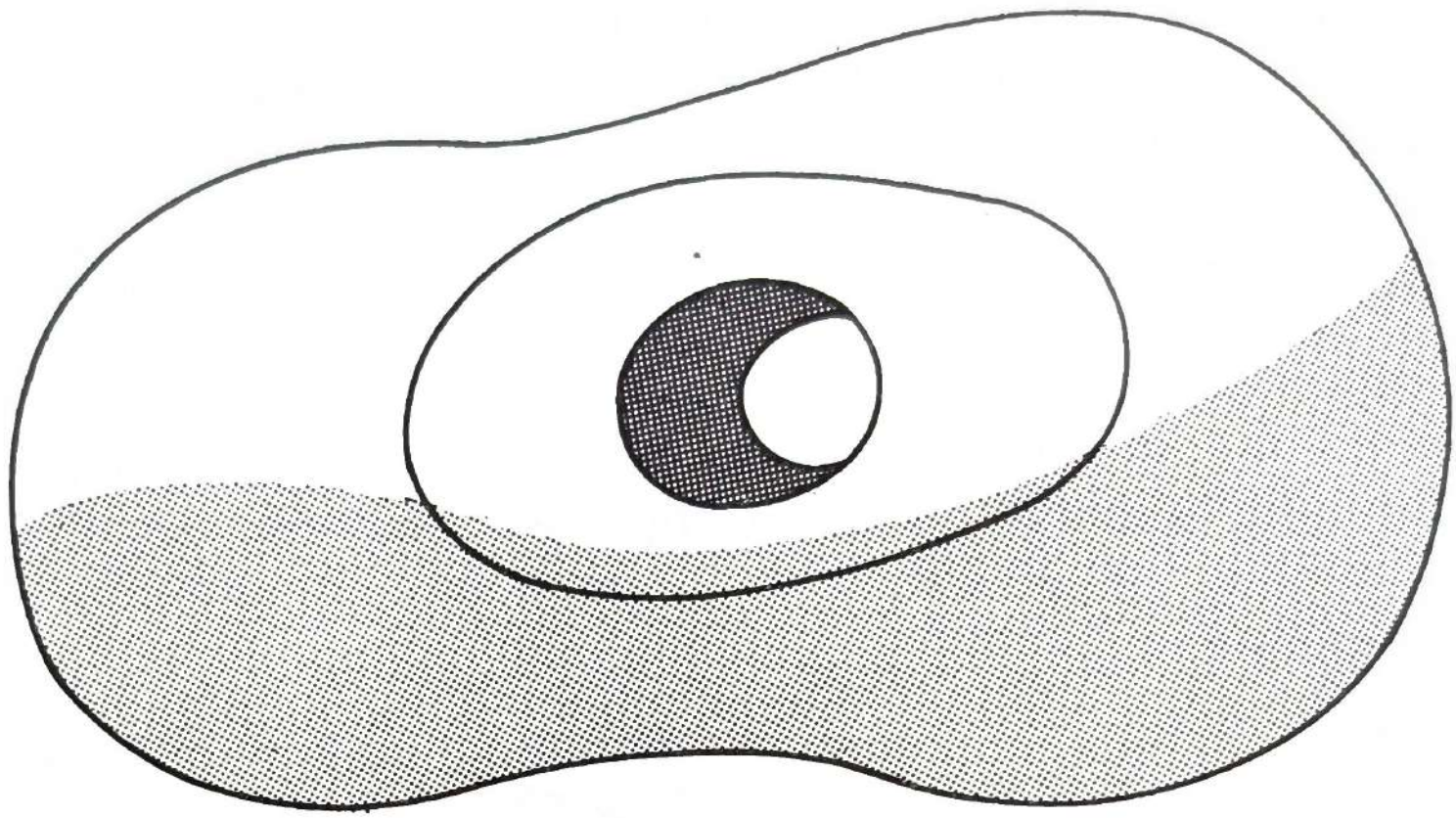
A great many things around us are alike in this way. How many can you discover? Our own bodies are also like this. A closed curve drawn on the skin forms two regions. It is very fortunate that this is so! Germs may be kept away from one part of the skin by enclosing it with a continuous boundary. This is really what we do when a cut is covered with a sterile dressing.

Can you imagine a shape on which a closed curve may be drawn that will not divide the surface up into two *separate* regions? At first, this may seem to be an impossible problem, but it is not so difficult as you might think. Take a piece of plasticine and mark a closed curve on it:

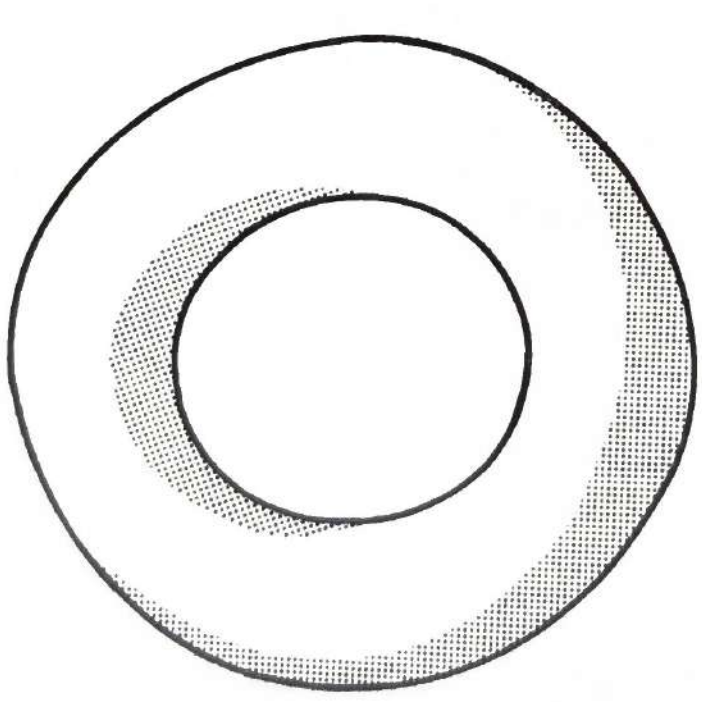


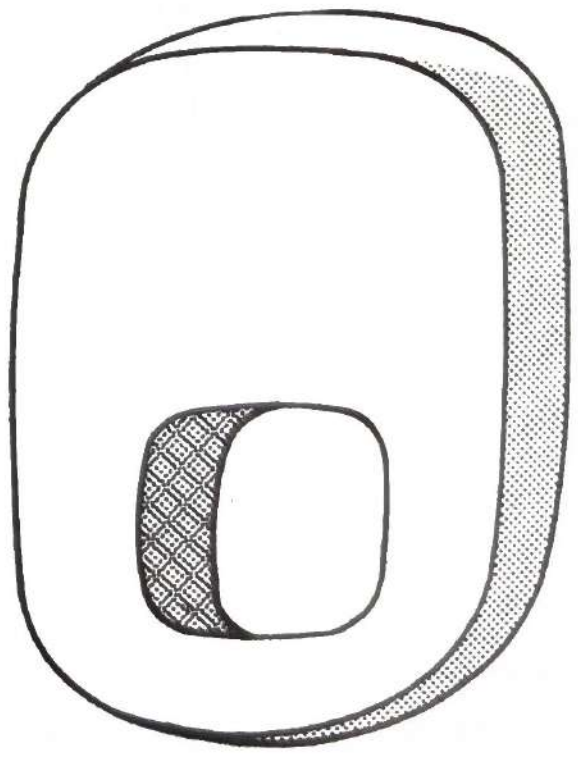
Any point inside the curve can only be reached from outside by crossing the boundary.

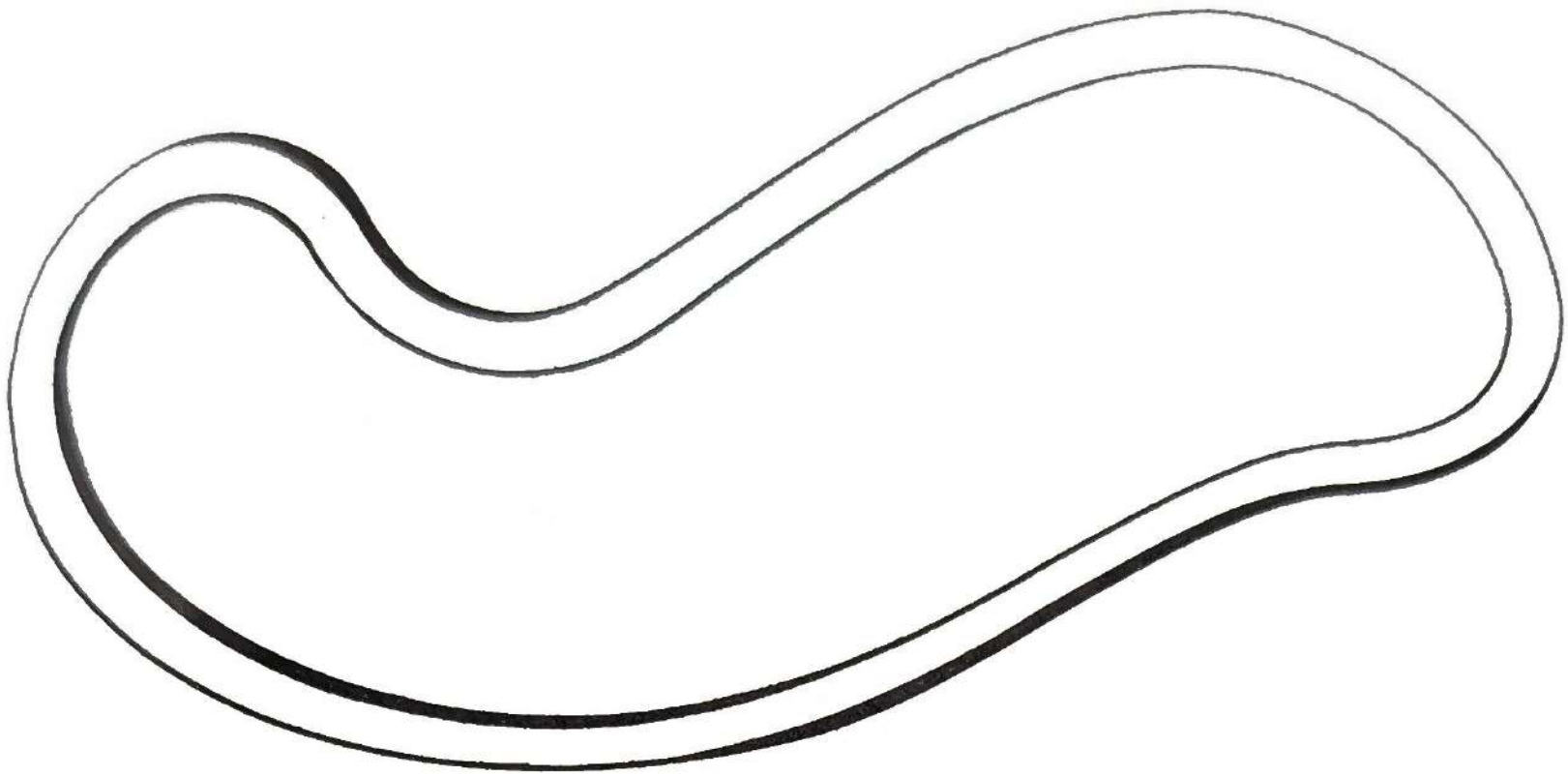
Now make a hole through the plasticine from inside the closed curve. The curve remains as before, but it no longer breaks up the surface into two regions! Any point inside the boundary can be reached from any point outside it by going through the hole!



The shape of the plasticine may be changed without altering this property:







Make some other shapes of this kind.

This chapter has helped you to look at shapes in new ways. You have learnt that certain properties of shapes remain the same even if the shapes are stretched and twisted. The study of this is a special kind of geometry called 'topology' and there is much more to find out.

Although you have now come to the end of the whole book, you are really only beginning to learn about the shapes of things. You must now see what you can observe and discover for yourself or with the help of other people.